AN INVESTIGATION OF THE COMPUTATION OF UPPER CONFIDENCE LEVELS IN A SERIES SYSTEM

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THESIS.

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by

Brent Dean Foster

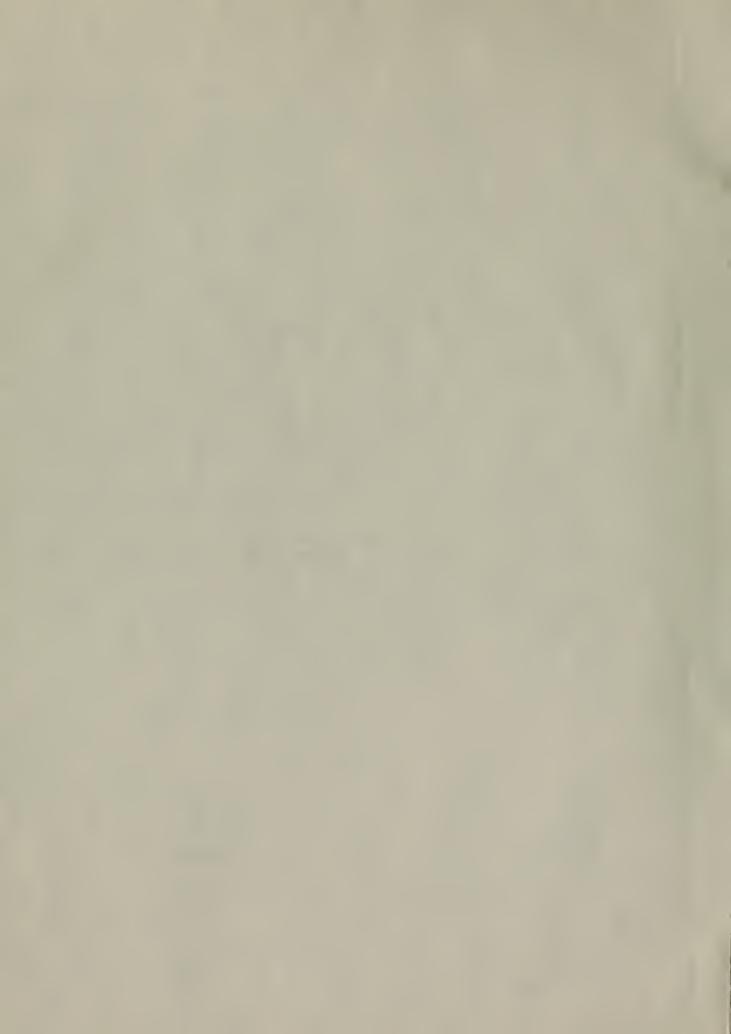
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An Investigation of the Computation of Upper Confidence Levels in a Series System

by

Brent Dean Foster Lieutenant Commander, United States Navy B.S., University of Wyoming, 1961

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ABSTRACT

A comparison of several techniques is presented for determining upper confidence levels for a system failure rate. A series system of components with exponential failure rates is examined. Classical computational techniques are compared with Bayesian techniques in determining the upper confidence level of a system failure rate. A sensitivity analysis is conducted on several of the parameters as part of the comparison.



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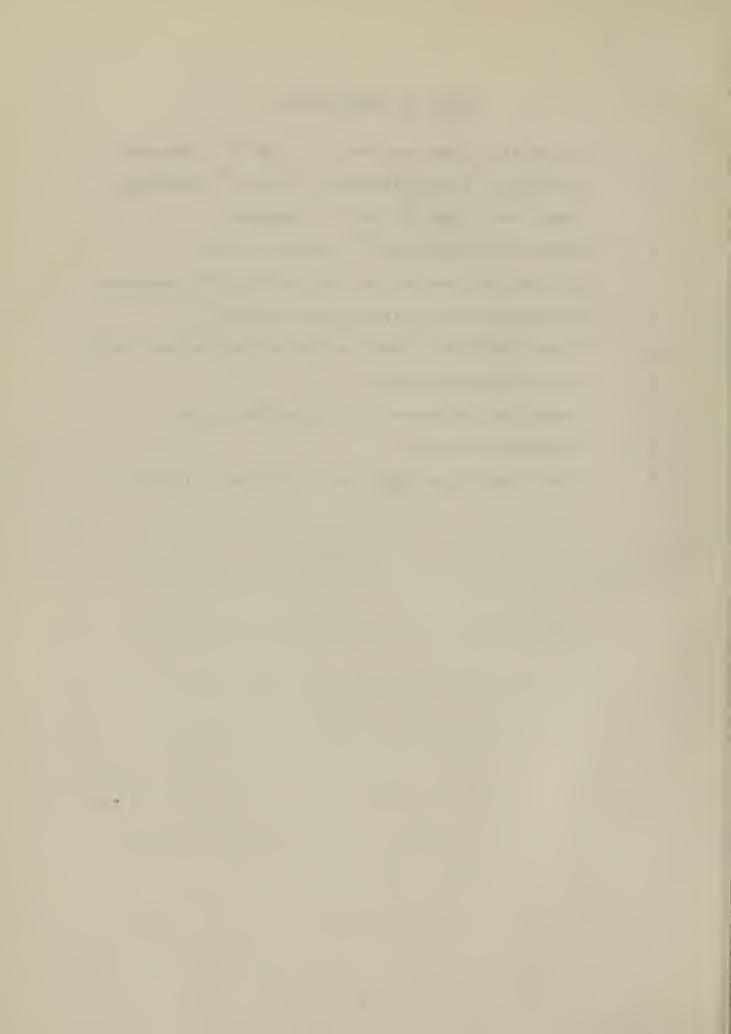
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TABLE OF ABBREVIATIONS

$\alpha_{\tilde{i}}$	Optimistic shape parameter of the i th component								
β _i	Optimistic scale parameter of the i th component								
ti	Total test time of the i th component								
fi	Number of times the i th component fails								
λ _i	Estimate of the failure rate of the i th component								
λ _s	Estimate of the system failure rate								
$\lambda_{\mathbf{u}}$	Upper confidence level on the system failure rate								
Υ	Level of significance								
k	Number of components in the series system								
R _s	System reliability								
R ₁	Lower confidence limit on the system reliability								



I. INTRODUCTION

A. BACKGROUND

Numerous "classic" techniques have been used to compute estimates of failure rate and mean time of failure. From these estimates standard accepted procedures can be applied to establish upper confidence levels (UCL) for the failure rate or lower confidence levels (LCL) for the reliability.

One well established "classic" procedure is to utilize the computational methods set forth in Ref. 3 to determine upper confidence levels on system failure rates. This is the procedure used for all "Classic" and "Semi-Classic" methods presented in this paper.

The application of a "Bayesian" technique may be intuitively appealing to some. Results from previous experiments and testing could be applied apriori to current testing programs to determine failure rate and reliability. By using a prior in such a manner perhaps total system test time, number of component failures, etc., could be reduced, thereby reducing the overall expense involved with system life testing. This would appear to be most appealing when testing systems that are extremely expensive.

The intent of this thesis is to attempt to determine if it would be more advantageous to use some "Bayesian" technique rather than a more traditional "classic" technique to compute upper confidence levels on a system failure rate



when the "prior" for the Bayesian method could be chosen as "optimistically" as one would desire.

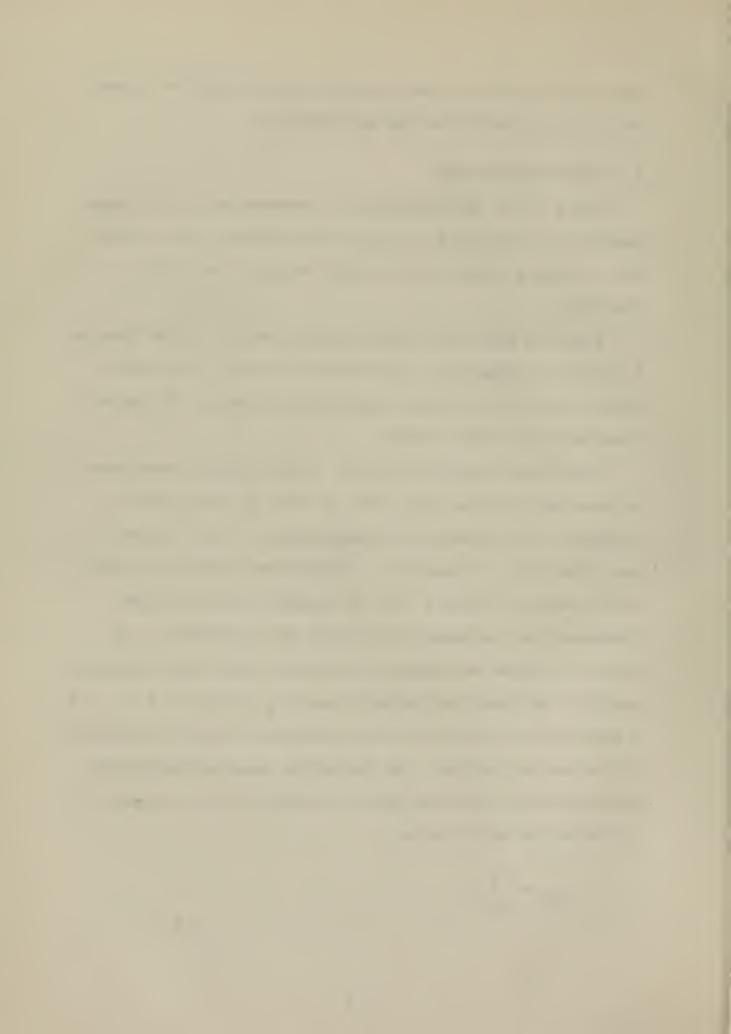
B. SYSTEM ASSUMPTIONS

Since it is the technique of computation of the upper confidence level that is being investigated, the system that is being modeled can be kept simple, yet still realistic.

Even the most complicated system can be broken down to a system of components, connected in series, which the total failure of any one component will result in the mission failure of the system.

Each type component in this series system experiences exponential failure rate. The failure of each type of component is assumed to be independent of the failure of any other type of component. Wearin and wearout are neglected and the failure rate is assumed to be constant. Components are assumed not to ever be "stillborn." To keep the system as simple as possible, each type component exhibits an identical failure rate, $\lambda_1 = \lambda_1$; $i = 2 \dots k$, k equaling the number of type components that are connected in the series system. The following formulas concerning system failure rate and system reliability are assumed to be valid for this system:

$$\lambda_s = \sum_{i=1}^k \lambda_i$$

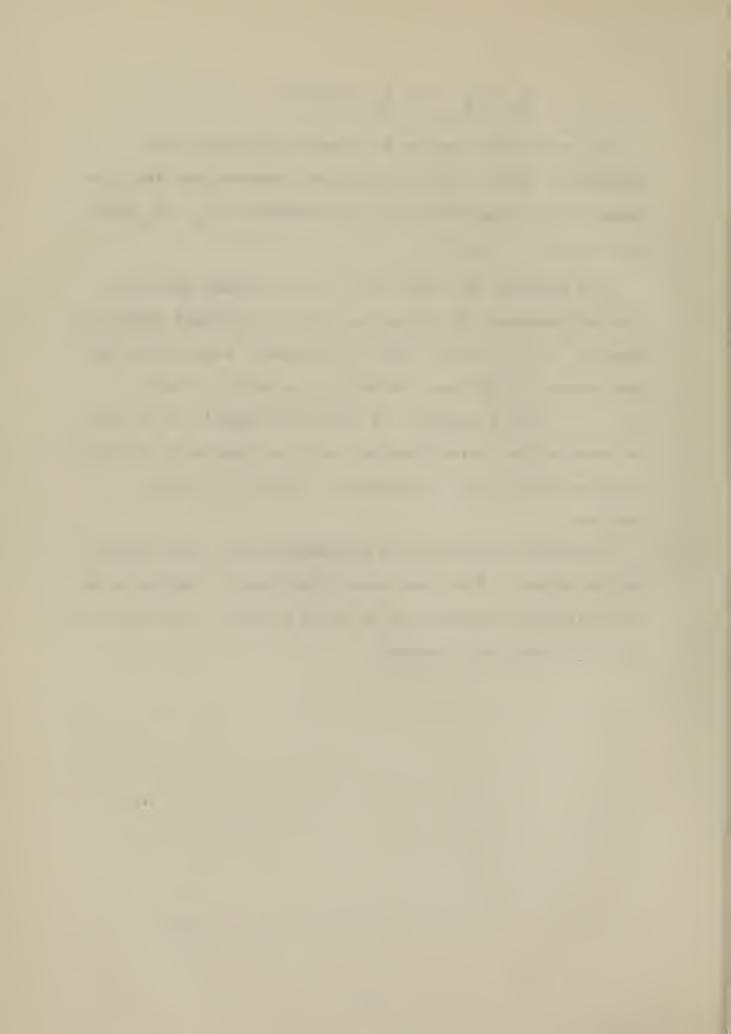


$$R_s = R_1$$
 or $R_s = \exp^{-\lambda}s$

In keeping the system as simple as possible the optimistic "prior" for the Bayesian computations are also chosen to be equal for each type component, $\alpha_1 = \alpha_i$ and $\beta_1 = \beta_i$, for $i = 1, 2, \ldots k$.

For purposes of comparison, it is assumed that each type of component is placed on test for an equal length of time, $t_1 = t_i$, and each type of component experiences the same number of failures during its total test time, $t_1 = t_i$. This assumption is modified slightly to be able to examine the system that has only one component failure. A system exhibiting two component failures is also examined.

The data (values of the parameters) have been chosen by the author. They have been intentionally chosen to be computationally simple yet to still exhibit the characteristics of a realistic system.



II. COMPUTATIONAL METHODS

A. CLASSIC

Reference 3 is used exclusively for the computations in this method. The computational formulas given in this reference are modified slightly to accommodate the basic assumptions of identical test time, identical number of failures for each component, etc.

 $\hat{\lambda}_i$ is a Maximum Likelihood Estimate for the failure rate of the ith component. It will be equal to the total number of failures divided by the total test time.

$$\hat{\lambda}_{i} = \frac{f_{i}}{t_{i}}.$$

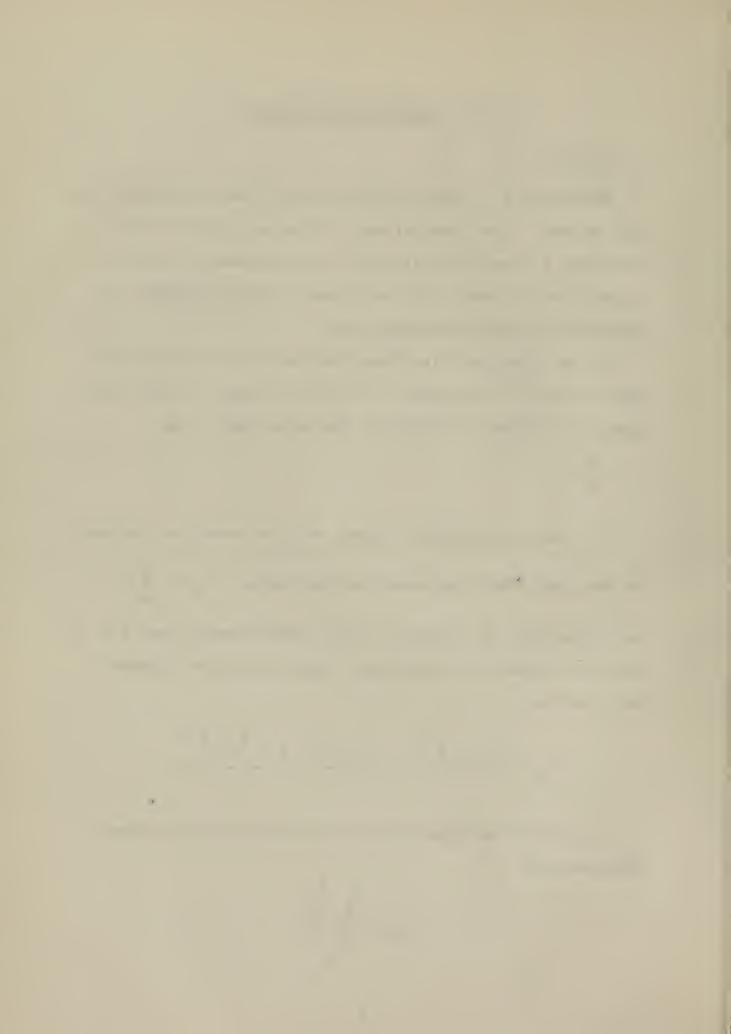
 $\hat{\lambda}_s$, the system failure rate, will be equal to the sum of the individual component failure rates. $\hat{\lambda}_s = \sum_{i=1}^k \hat{\lambda}_i$.

Each component is assumed to fail independently and k w: 1 equal the number of components that are placed together in a series.

$$\lambda_{\rm u} = \frac{2\hat{\lambda}_{\rm S} + {\rm K'}^2{\rm C} + (4\hat{\lambda}_{\rm S}{\rm K'}^2{\rm C} + {\rm K'}^4{\rm C}^2)^{1/2}}{2}$$

 $\boldsymbol{\lambda}_{u}$ will be an upper confidence level on the system failure rate.

$$C = \frac{\sum_{i=1}^{k} \frac{\hat{\lambda}_{i}}{t_{i}}}{\hat{\lambda}_{s}}$$



The values for K' are found in Table I. The computational procedure to determine the Beta values for a 90 per cent level of significance are also discussed. Eighty per cent level of significance values are found in Ref. 3.

A formula for the upper confidence level for the system failure rate is also provided when no failures have occurred during the total test time that each component has been allotted. That formula follows:

$$\lambda_{u} = \frac{K'^{2}}{n} \sum_{i=1}^{k} \left(\frac{1}{t_{i}}\right)$$

where n equals the number of component terms in the summation.

Substitution of the upper limits on the failure rates obtained will generate corresponding lower confidence limits on reliability; thus

$$R_1 = \exp^{-\lambda} u$$
.

Beta can be calculated from the following formula:

$$X_u - X = Beta(K)(X_u)^{1/2}$$

 $X_{\rm u}$ is obtained from Chi-Squared tables in Ref. 1 at the desired confidence level with 2(X+1) degrees of freedom where X is the number of failures and K is the percentage point of the normal distribution point in the same confidence level.



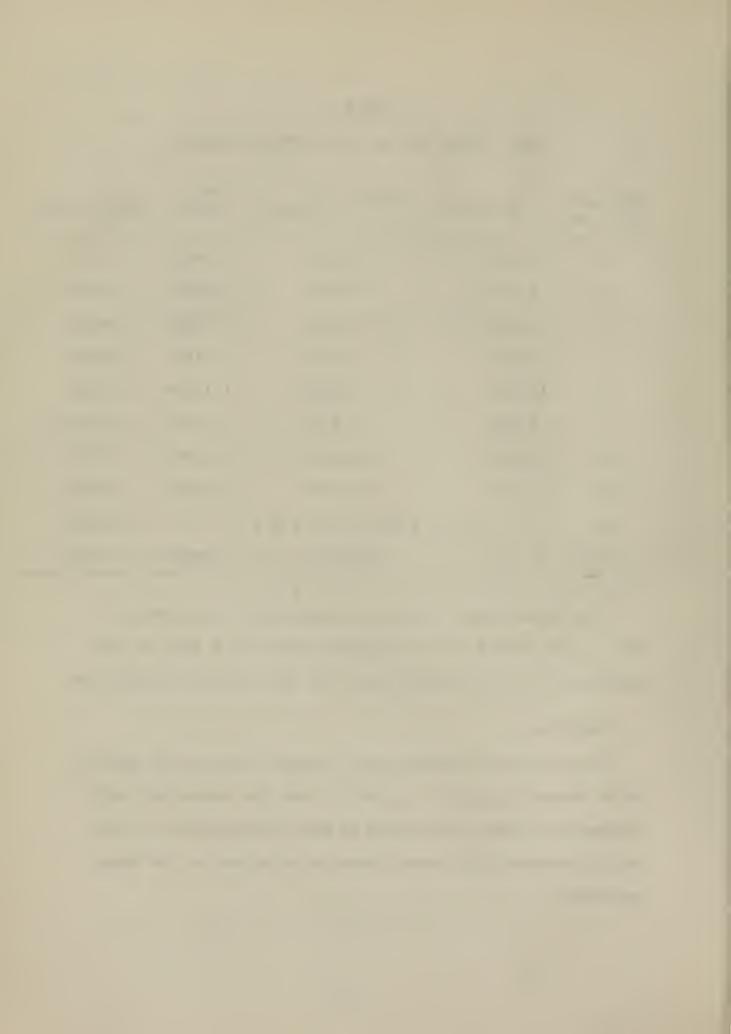
TABLE I
BETA VALUES FOR 90 PER CENT CONFIDENCE

X No. of Failures	x ² .90,2(X+1)	$(1/2X^2 X_{\mu}^{\chi}, 90, 2(X+1))$	Beta Value	K' (Beta(1.282))
0	4.906	2.3025	1.18362	1.5174
1	7.779	3.889	1.14271	1.46496
2	10.645	5.3225	1.12336	1.44015
3	13.362	6.681	1.1109	1.42417
4	15.987	7.994	1.10188	1.41261
5	18.549	9.275	1.09494	1.403713
10	30.813	15.406	1.0743	1.37725
20	54.090	27.045	1.05669	1.35468
30	I N T	E R P O L.A T E D		1.34639
36.5	91.1	45.55	1.04596	1.34092

The "Beta Value" listed in this table is germane to Ref. 3 and should not be confused with the β that is the optimistic scale parameter used in the Bayesian simulation.

B. BAYESIAN

The Bayesian technique will assume the apriori density to be $\operatorname{Gamma}(\lambda_i;\alpha_i,\beta_i)$. α_i and β_i are the shape and scale parameters. The "prior" can be made "optimistic" if the scale parameter is chosen large in relation to the shape parameter.



The a posteriori density then becomes $(\lambda_i; \alpha_i + f_i, \beta_i + t_i)$. f_i will equal the number of failures of the ith type component and t_i will equal the total test time for the ith type component.

The distribution of $\lambda_s = \sum_{i=1}^{k} \lambda_i$ will then be determined

by computer simulation. Random variates of each λ_i are generated from the Gamma distribution with parameters $\alpha_i + f_i$, $\beta_i + t_i$. A random variate will be generated for each λ_i , $i = 1, \ldots, k$, the number of components in series. The λ_i 's will then be added to determine the series system failure rate.

The process for generating a system failure rate is then repeated 1000 times to yield λ_{s1} , λ_{s2} , . . . λ_{s1000} . The 1000 random values of λ_{s} are then ordered to yield $\lambda_{s}(1)$, $\lambda_{s}(2)$, . . . $\lambda_{s}(1000)$.

 $^{\lambda}su(\gamma)$ is the "estimate" of the (1 - $\gamma)^{\mbox{th}}$ percentile point of the distribution of λ_s .

The Bayesian $100(1 - \gamma)^{th}$ upper confidence level for λ_s is then $\lambda_{s1000}(1 - \gamma)$.

At the time of this writing a subroutine to generate random variates from the Gamma distribution does not exist in the Naval Postgraduate School computer library. Reference 2 was used to generate random variates from the Gamma distribution with an integer shape parameter.



The generation of random variates with a shape parameter that is not an integer poses a much more complicated problem. Reference 4 has been written to handle this situation for shape parameters between 0.05 and 1.0. Although LT Robinson's approach is relatively untried, it shows a great deal of promise and may possibly be incorporated into the Naval Postgraduate School computer library in the future. It has been used in this writing for comparative purposes for a shape parameter less than 1.0.

C. SEMI-CLASSIC

Similar to the Classic technique, Ref. 3 is used to compute the upper confidence levels for the system failure rate for this method. The computations will be modified somewhat by adding the identical "optimistic" priors used in the Bayesian technique.

The number of failures per component will be added to α_i and the total test time per component will be added to β_i . A maximum Likelihood Estimate for the ith component's failure is then:

$$\hat{\lambda}_{i} = \frac{f_{i} + \alpha_{i}}{t_{i} + \beta_{i}}$$

The system failure rate for this Semi-Classic method is the same as for the Classic method.

$$\hat{\lambda}_{s} = \sum_{i=1}^{k} \hat{\lambda}_{i}$$



The value of C will also change slightly:

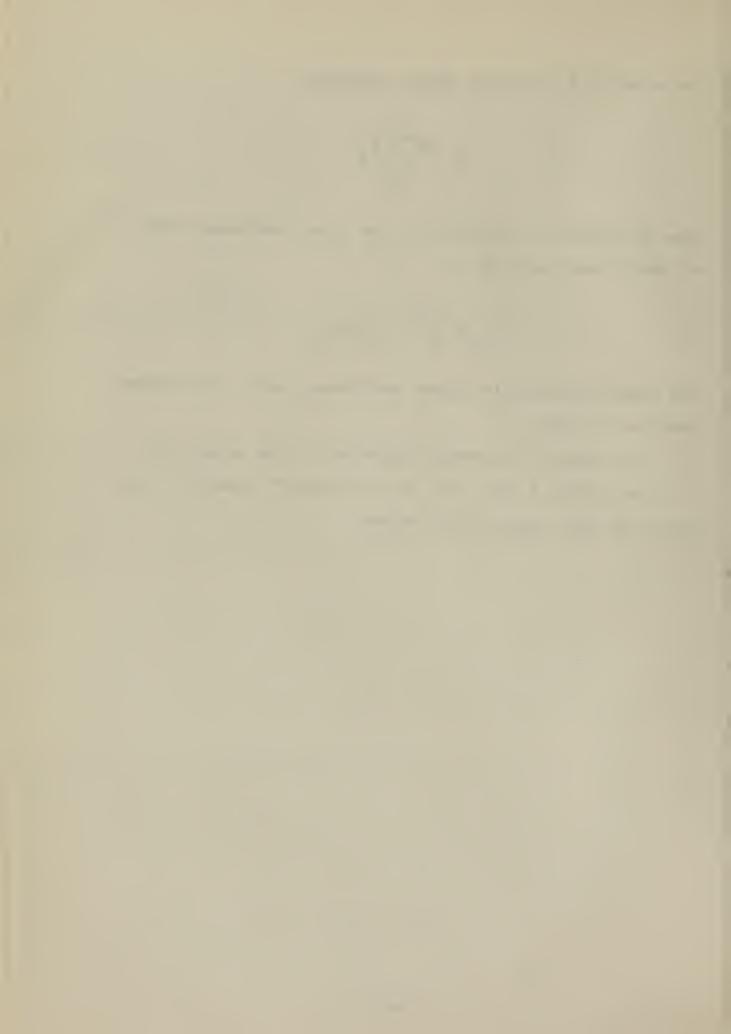
$$C = \frac{\hat{\lambda}_{i}}{\frac{t_{i} + \beta_{i}}{\hat{\lambda}_{s}}}$$

When no failures have occurred, the upper confidence limit on the failure rate now is:

$$\lambda_{u} = \frac{K^{2}}{n} \frac{1}{t_{i} + \beta_{i}}$$

The computation for the upper confidence level with failure remains the same.

This method is somewhat "Bayesian" in the sense that it is utilizing a prior but it is "Classic" because of the nature of the computational method.



III. PARAMETERS OF VARIATION

Although the range through which some of the parameters are varied could be more extensive, it does provide the reader with an adequate base for comparing the methods of computation.

- α_{i} 1.0, 0.5
- β_{i} 50, 000
- t_i 30, 50, 100 mission units
- f_i 1.0, 0.0 The failures are also modified so that $f_1 = 1$, $f_2 = f_i = 0$ and $f_1 = 1$, $f_2 = 1$, $f_3 = f_i$, $f_i = 0$. This will provide the reader with the opportunity to compare a system where only one component or only two components fail.
- k 1, 2, 5, 10, 30 components
- γ 0.10



IV. COMPARISONS OF SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS

The following 12 tables provide the reader with upper confidence levels computed from the same data by the three different methods: Classic, Semi-Classic, and Bayesian.

A program to generate random variates from a Gamma distribution with a shape parameter that was greater than 1.0 but not an integer was not available; therefore UCL's for the Bayesian simulation could not be computed for the case when the shape parameter was less than 1.0 and a failure existed for the ith component.

All calculations were conducted on the IBM 360 computer with the exception of the Classic method with zero failures, whose calculations were computed on a desk calculator.

For each three-line block of numbers the reader may compare the three methods' upper confidence levels computed from the same arguments. To see how a change in the scale parameter, the number of times a component fails, or the number of components in series is reflected in the upper confidence level, the reader merely moves to the right or down in the table. If the reader wants to see how a change in the shape parameter or a change in the amount of time a component is on test is reflected, he must go to another table.

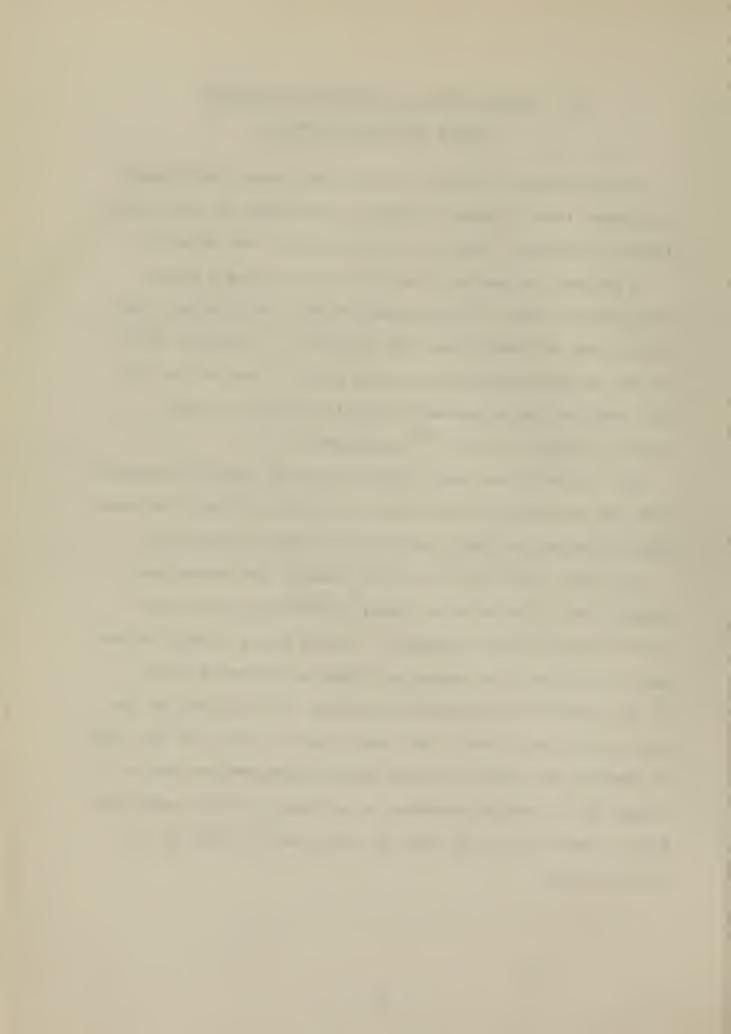


TABLE II

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS ALFA; = 1.0	2 5 10 30	.076750 .076750 .076750	.066531 .115937 .192571 .478729	.046493 .097500 .172700 .460052	.174417 .309166 .513524 1.276610	.101229 .339753 .891556	.083120 .321967 .868478		.076750 .076750 .076750	.040942 .071346 .118505 .294603	.028611 .060000 .106277 .283109		.174417 .309166 .513524 1.276610	.062295 .119478 .209079 .548650	.051577 .106277 .198133 .534447
SYS	∀	CLASSIC .076750	SEMI .048790 CLASSIC	BAYESIAN026661	CLASSIC .130107	SEMI .067770	BAYESIAM .046493		CLASSIC .076750	SEMI .030025 CLASSIC	BAYESIAN .016407		CLASSIC .130107	SEMI .041705	BAYESIAN .028611
	Beta _j Fail _j		0		09	. –				0		8			
	Time							8							



TABLE III

.713245 .046050 255322 .463188 046050 382984 765967 694782 765967 475497 368041 245361 30 = 1.0 SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS ALFA; .257573 046050 .102705 .181202 .171715 .271802 308114 .046050 .154057 .138160 .308114 .092107 10 .185500 .103548 .061833 .052000 .046050 .092750 .078000 .185500 .138160 .046050 .092107 .155321 S .024796 .046050 .053225 .080983 .046050 .035483 .106450 .053989 .044700 .106450 067050 .037194 2 .054216 .039032 .024796 .014219 .046050 .021329 .078064 .037194 046050 .078064 .036144 .026021 × BAYESIAN BAYESIAN BAYESIAN BAYESIAN SEMI CLASSIC SEMI CLASSIC SEMI CLASSIC SEMI CLASSIC CLASSIC CLASSIC **CLASSIC** CLASSIC Fail; 0 0 Beta; 20 100 Time, 50

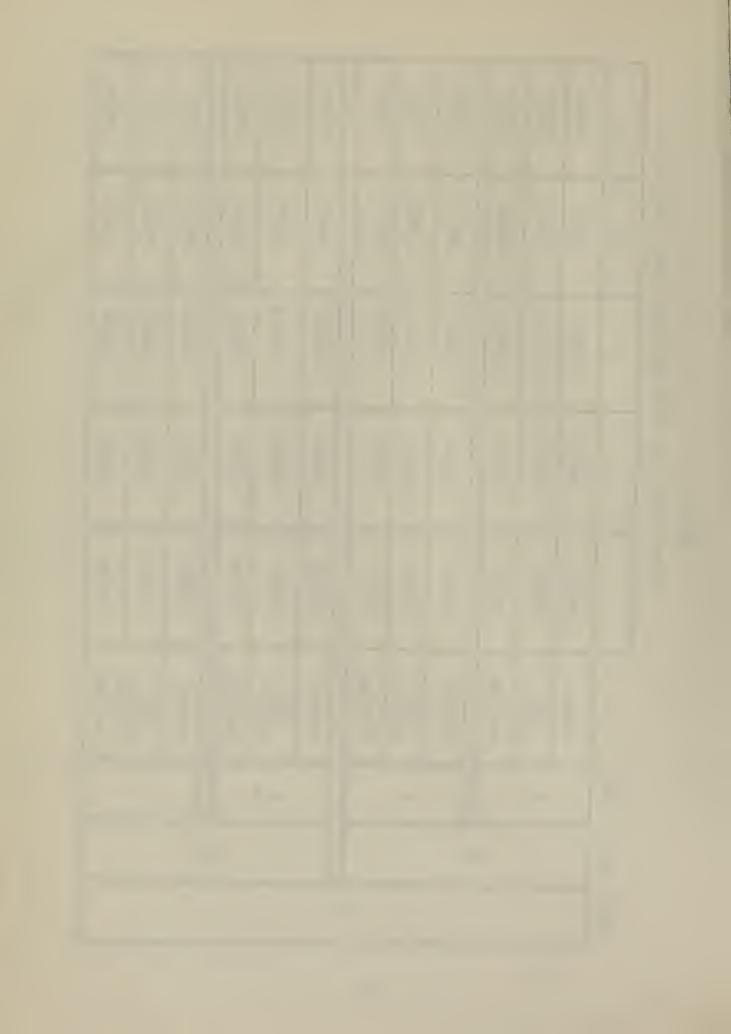


TABLE IV

.023025 356623 023025 255322 .463188 .191492 382984 .347391 382984 475497 .184021 .245361 30 = 1.0 ALFA; .154057 .181202 .102705 .171715 .077029 080690 .135901 .023025 .023025 .154057 .092107 .128787 SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS 9 .046375 .069080 .052000 .103548 .077661 .061833 .092750 .023025 039000 023025 .092750 .092107 2 .024796 .023025 .035483 .053225 .053989 .023025 .026612 .053225 .033525 .044700 .040492 .018597 2 .014219 .024796 .023025 .019516 .027108 .023025 .010665 .039032 .026021 .039032 036144 .018597 ¥ BAYESIAN BAYESIAN BAYESIAN SEMI BAYESIAN SENI CLASSIC SEMI CLASSIC CLASSIC CLASSIC CLASSIC CLASSIC CLASSIC SEMI Fail 0 0 Beta; 20 100 Time, 100

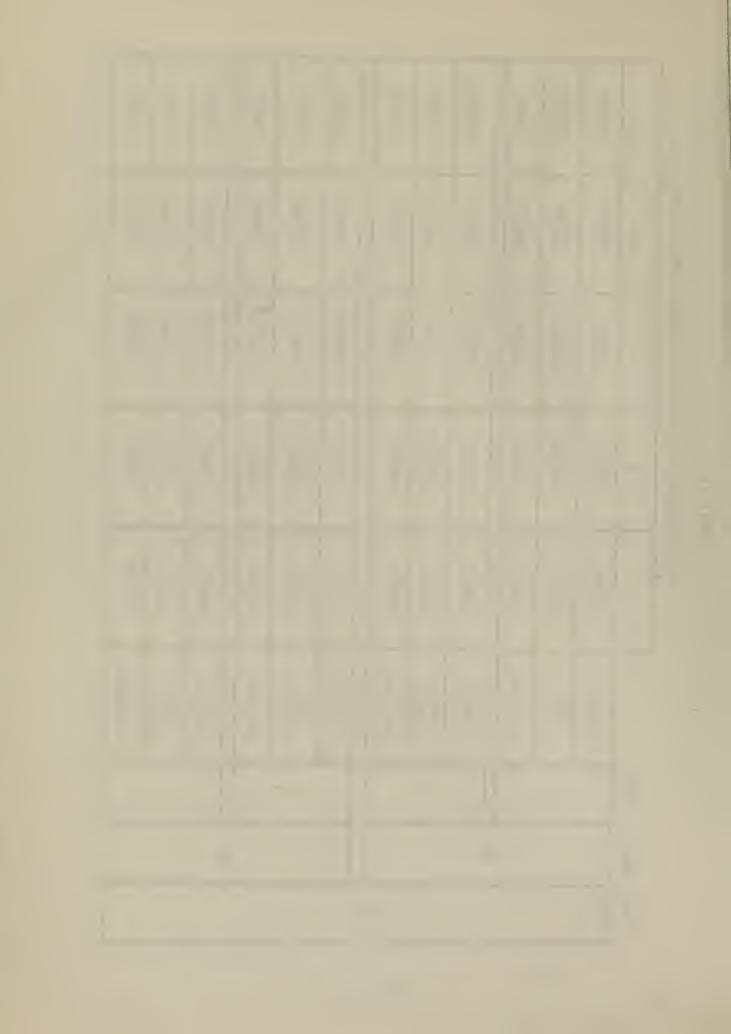


TABLE V

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS ALFA; = 0.5	1 2 5 10 30	.076750 .076750 .076750 .076750	.038477 .047646 .073919 .114634 .264620	.018439 .028627 .057292 .101570 .258735		.130107 .177417 .309166 .513524 1.276610	.058481 .084230 .155671 .686737	******		.076750 .076750 .076750 .076750	.023678 .029321 .045488 .070544 .162843	.011347 .017617 .035257 .062505 .159222		.130107 .177417 .309166 .513524 1.276610	.035988 .051834 .095797 .164355 .422608	******* ****** ****** ******
	 	CLASSIÇ	SEMI CLASSIC	BAYESIAN ·		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN
	Fail		0				_				0				_	
	Betai				20								100			
	Time,							Č	30							



TABLE VI

= 0.5	30	.046050	.211696	. 206988		.765967	.549390	*****		.046050	.141131	.137992		.765967	.366260	*****
ALFA ₁	10	.046050	.091708	.081256		.308114	.213661	*****		.046050	.061138	.054171		.308114	.142441	*****
PER CONFIDENCE	വ	.046050	.059135	.045834		.185500	.124537	*****	l	.046050	.039423	.030556		.185500	.083024	*****
SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS	2	.046050	.038117	.022902		.106450	.067384	****		.046050	.025411	.015268		.106450	.044923	*****
SYSTEM	1	.046050	.030078	.014751		.078064	.046785	*****		.046050	.020521	.009834		.078064	031190	*
	∠	CLASSIC	SEMI CLASSIC	BAYESIAN .		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN
	Faili		0				<u>-</u>				0				<u> </u>	
	Beta;				20								100			
	Time,								50					-		



TABLE VII

.023025 .105848 274695 ***** .137992 382984 366260 ***** .103494 382984 023025 .141131 30 0.5 11 SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS ALFA; .040628 ***** .023025 ***** .061138 .045854 023025 .054171 .154057 .142441 .154057 .106831 9 .030556 062268 ***** ***** .023025 .029568 .022917 .092750 .023025 .039423 .092750 .083024 2 .025411 .015268 .053225 .044923 ***** .023025 .019058 .053225 .033692 ***** 023025 .011451 2 .007376 .023025 .009834 .031190 ***** .023025 .039032 .023390 ***** .039032 .020521 .015391 ¥ BAYESIAN BAYESIAN **BAYESIAN** BAYESIAN SEMI CLASSIC SEMI CLASSIC SEMI CLASSIC SEMI CLASSIC CLASSIC CLASSIC CLASSIC CLASSIC Faili 0 0 Beta, 100 50 Time; 100



TABLE VIII



TABLE IX

			MODIFIED UPPE	MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES	EVELS ON SYSTE	M FAILURE RATES	$ALFA_{i} = 1.0$
Faili	•-		1	2	5	10	30
f ₁ = 1		CLASSIC	.078064	.078064	.078064	.078064	.078064
f ₂ = f	·-	SEMI CLASSIC	.054216	.093570	.136890	.203823	.449300
f.= 0		BAYESIAN	.037194	.052215	109160.	.150216	.377801
-L		CLASSIC	*****	.106450	.106450	.106450	.106450
$f_2 = 1$ $f_3 = f$. *	SEMI CLASSIC	*****	.106450	.148479	.214278	.457804
ر 1 = 0	-	BAYESIAN	*****	.067050	.103548	.162349	.391324
f,= 1		CLASSIC	.078064	.078064	.078064	.078064	.078064
		SEMI CLASSIC	.036144	.078064	.108433	.155206	.325173
- i -		BAYESIAN	.024796	.034810	.061067	.100144	.251867
f ₁ = 1		CLASSIC	*****	.106450	.106450	.106450	.106450
f2= 1		SEMI CLASSIC	****	. 086603	.116048	.161966	.330403
ر ا=أ-		BAYESIAN	*****	.044700	.069032	.108232	.260883
	1						

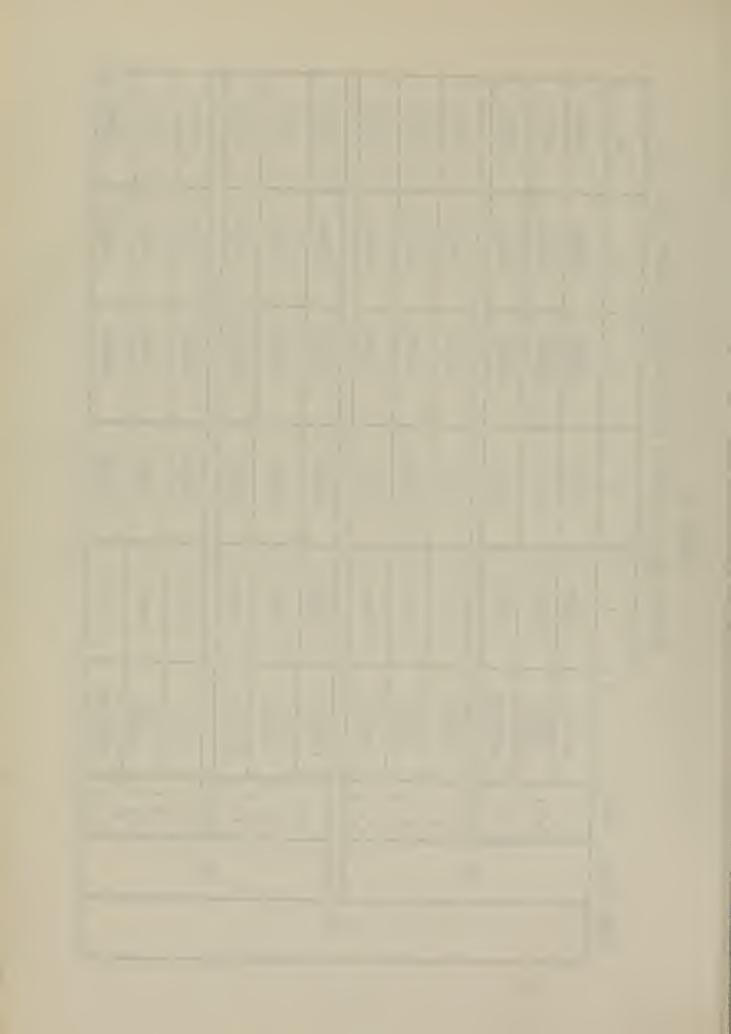


TABLE X

M FAILURE RATES ALFA; = 1.0	10 30	.039032 .039032	.125363	.100144 .251867		.053225 .053225	.132403 .291025	.108232 .260883		.039032 .039032	.101911	.075108 .188900	-		.053225 .053225	
MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES	5	.039032	.082107	.061067		.053225	088830	.069032		.039032	.068445	.045800		.053225		.074240
ER CONFIDENCE L	2	.039032	.054216	.034810		.053225	.062739	.044700		.039032	.046785	.026107		.053225		.053225
MODIFIED UPPE	1	.039032	.036144	.024796		*****	*****	*****		.039032	.027108	.018597		*****		****
٠.	~	CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC		SEMI CLASSIC
	Fail	f ₁ = 1	f ₂ = f ₁	f ₁ = 0		f_= 1	$f_{2} = 1$	ر د f _j = 0		f ₁ = 1	f2= f;	-	4		-	f2= 1 f3= f;
	Betaj				20							00'	201			
	Time,							I	100							



TABLE XI

RE RATES ALFA; = 0.5	10 30	.130107	.192806 .361256	*****		.177417 .177417	. 207180	*****		.130107 .130107	.150096 .259836	*****		.177417	.158538 .266507	*****
M FAILU		-	-	*		Ţ.	.2	*		-	١.			<u>-</u> .	Г. 	*
EVELS ON SYSTE	Ŋ	.130107	.146407	****		.177417	.162117	****		.130107	.119775	*****		.177417	.129082	****
MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES	2	.130107	.116621	*****		.177417	.133609	****		.130107	.100392	*****		.177417	.110467	****
MODIFIED UPPE	1	.130107	.054810	*****		****	****	****		.130107	.035988	*****		****	****	****
•		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN		CLASSIC	SEMI CLASSIC	BAYESIAN
	Fail	f ₁ = 1	$f_2 = f_1$	f.= 0		f ₁ = 1	$f_2 = 1$ $f_3 = f_3$	f = 0		f,=]		†= 0	I	f ₁ = 1	$f_2 = 1$ $f_3 = f_4$	f _. = 0
	Betai				20								100			
	Time,								30	3						



TABLE XII

		-	MODIFIED UPPE	CONFIDENCE LE	MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES	FAILURE RATES	$ALFA_{i} = 0.5$
Fail		×	1	2	5	10	30
f_= 1		CLASSIC	.078064	.078064	.078064	.078064	.078064
f2= f;		SEMI CLASSIC	.046785	.078064	. 101068	.136890	.267480
0 # +-		BAYESIAN	*****	*****	*****	*****	*****
	- 1						
f ₁ = 1		CLASSIC	*****	.106450	.106450	.106450	.106450
f 2= 3= f,		SEMI CLASSIC	*****	.091662	.113665	.148479	.277238
f ₁ = 0		BAYESIAN	*****	****	*****	*****	*****
f ₁ = 1		CLASSIC	.078064	.078064	.078064	.078064	.078064
		SEMI CLASSIC	031190	.067210	.083321	.108433	.199486
- - -		BAYESIAN	*****	*****	*****	*****	****
f ₁ = 1		CLASSIC	*****	.106450	.106450	.106450	.106450
$f_2 = 1$		SEMI CLASSIC	*****	.076234	.091662	.116048	.205707
f.= 0		BAYESIAN	****	****	****	*****	*****
	1						

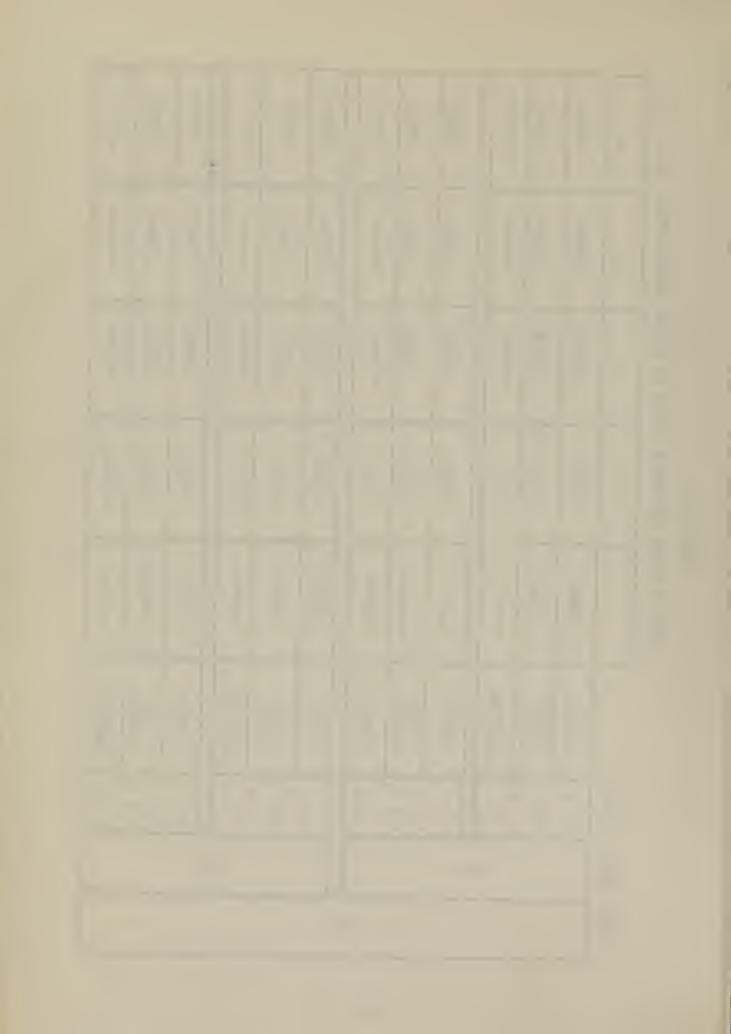


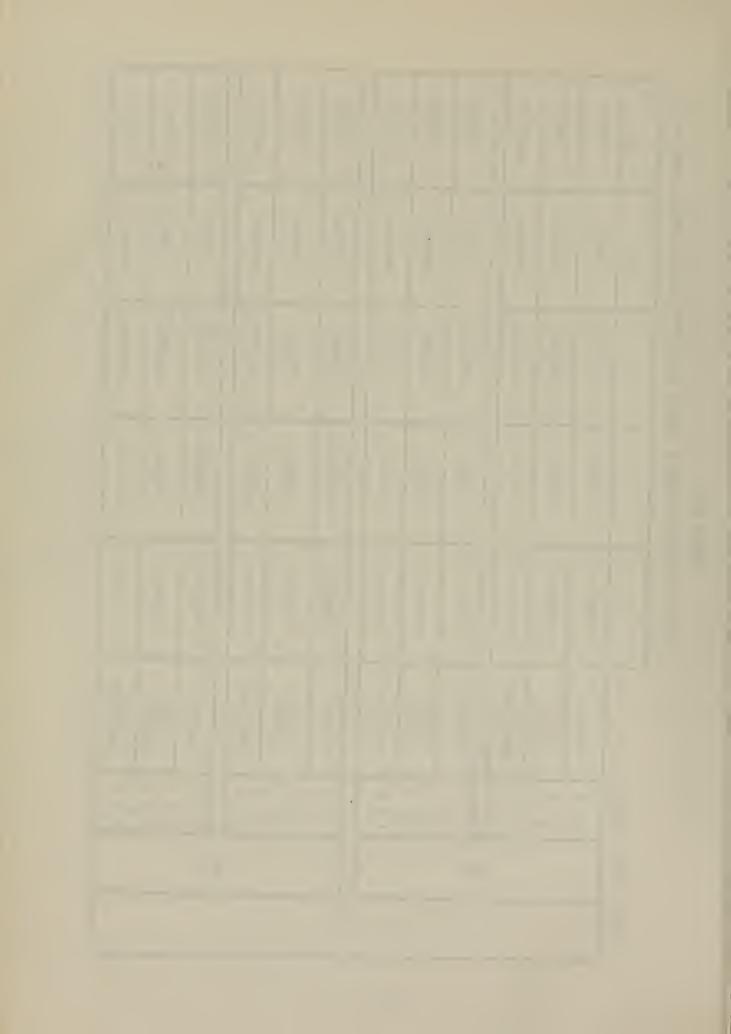
TABLE XIII

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES

0.5

ALFA =

.138619 .053225 .173283 ***** .039032 .133740 ***** 039032 ***** .053225 **** .166657 30 ***** .068445 ***** .053225 074240 .039032 ***** .053225 089830 .039032 .082107 **** 9 .053225 **** ***** .059039 ***** .053225 .067384 ***** .039032 .050534 .056833 039032 2 ***** .039032 053225 .045831 .044243 ***** .053225 .053225 ***** .039032 ***** .039032 2 **** ***** .023392 ***** ***** ***** ***** ***** ***** .039032 .039032 .031190 ¥ BAYESIAN BAYESIAN BAYESIAN BAYESIAN SEMI CLASSIC SEMI CLASSIC CLASSIC CLASSIC CLASSIC CLASSIC CLASSIC CLASSIC SEMI SEMI Fail; f.= 0 f = 0 0 f3= f2= f2= f2= f₂= Beta; 20 100 100 Time,



V. CONCLUSIONS

A "crossover" point in this discussion will be defined as the point or general area at which an upper confidence level that was previously lower than one with which it was being compared becomes higher.

The Bayesian method will be compared with the Classic and the Semi-Classic will be compared with Classic. The Bayesian and the Semi-Classic appear to behave similarly and remarks concerning this similarity will be mentioned.

A. UNMODIFIED COMPARISONS, ALFA = 1.0

The first three tables deal with a shape parameter of 1.0 and a system of components that either all of the components experience a failure or none of the components experience a failure.

In the cases where none of the components fail, a crossover point is exhibited in every case when five or more components are in series, regardless of how optimistic the scale parameter becomes or how long the test time is extended. When every component fails, the Bayesian and Semi-Classic systems do not crossover. In both cases the values of the upper confidence level for the Bayesian and the Semi-Classic methods become closer together as the number of components increases.



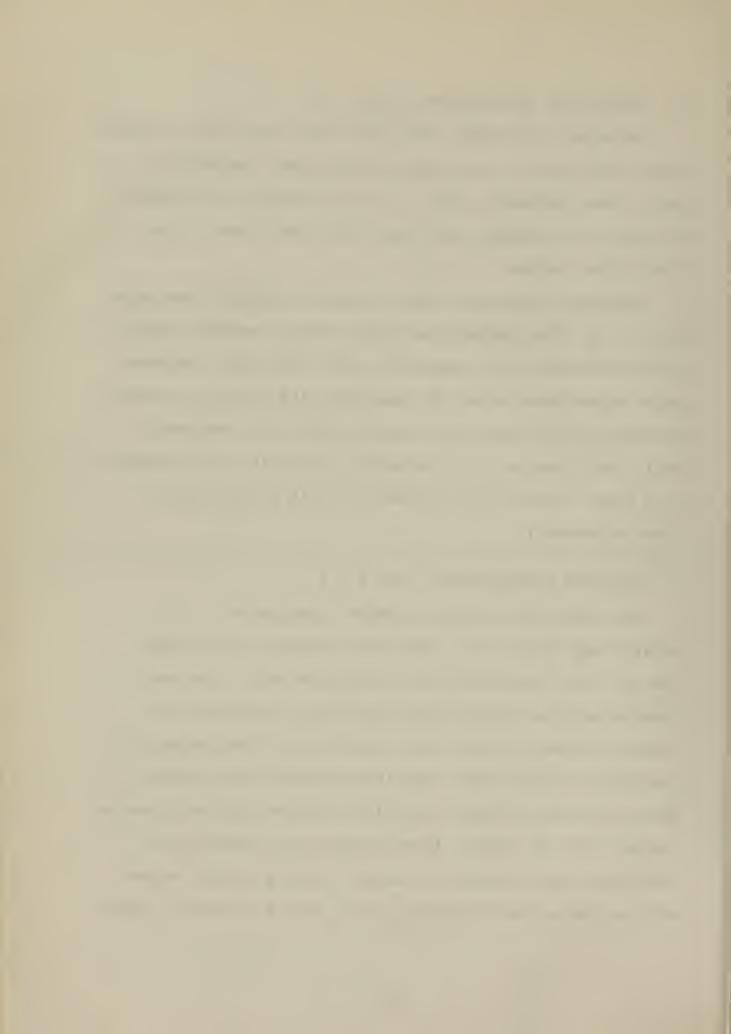
B. UNMODIFIED COMPUTATIONS, ALFA = 0.5

The upper confidence levels for the second three tables (less the Classic) are computed with a more optimistic prior shape parameter (alfa = 0.5) so naturally the UCL's for these two methods are lower than those shown in the first three tables.

Crossover points are much the same as in the case when alfa = 1.0. The Bayesian and Semi-Classic methods cross-over when none of the components fail; only the crossover point occurs when about 10 components are placed in series. The Semi-Classic does not crossover when all components fail (the Bayesian is not examined due to the non-existance of a Gamma random variate generator with a non-integer shape parameter).

C. MODIFIED COMPUTATIONS, ALFA = 1.0

The third three tables provide a much more interesting comparison. Here the situation where only one (or two) component(s) in the system fail. The Semi-Classic and the Bayesian techniques will crossover the Classic system in every case regardless of how optimistic the "priors" are or how long the components are tested. When the priors are more optimistic and/or the test time is larger, then it takes a larger number of components in series for the crossover to occur. This point may occur with as few as two components or as many as between 10 and



30 components. The Bayesian UCL's are strictly lower than the Semi-Classic UCL's.

D. MODIFIED COMPUTATIONS, ALFA = 0.5

In the last three tables only the Semi-Classic and the Classic methods can be compared for the Bayesian cannot be simulated for this case (non-integer).

The Semi-Classic technique is using a more optimistic prior so its UCL's are lower than before but again in every case it will crossover with the Classic method.

Generally one could conclude that as long as the number of components that are in series in a system are few, the Bayesian approach may yield a lower value of an upper confidence level for the system failure rate, provided the priors can be chosen optimistically. If the system is complex enough that more than "a few" components must be placed in series then the Classic system appears to yield the lowest values of upper confidence levels. The Semi-Classic technique would only be useful if the priors were optimistic, the number of components in series were few, and the optimistic priors were non-integers.



COMPUTER PROGRAM FOR THE CLASSIC METHOD

```
THE VARIABLE DEFINITIONS FOR THIS PROGRAM FOLLOW:

TIME THE NUMBER OF MISSION UNITS EACH COMPON-
ENT IS UNDER TEST

FAIL THE NUMBER OF TIMES THAT EACH COMPONENT
WILL FAIL DURING THE TOTAL TEST TIME

XK THE NUMBER OF COMPONENTS THAT ARE IN THE
SERIES SYSTEM
XLAMI INDIVIDUAL COMPONENT FAILURE RATE
XLAMS SYSTEM FAILURE RATE
XKPRI A VALUE TAKEN FROM TABLE I (BETA VALUES)
XK=1.0
XKPRI=1.469496
TIME=30.0
7
10
                        TIME=30.0

C=0.0

FAIL=1.0

XLAMUP=0.0

XLAMI=FAIL/TIME

XLAMS=XK*XLAMI

C=(XK*(XLAMI/TIME))/XLAMS

XLAMUP=(2.0*XLAMS+(XKPRI**2)*C+SORT(((4.0*XLAMS)*(XKPR

1I**2)*C)+(XKPRI**4)*(**2))/2.0

WRITE(6.101)FAIL,XK.TIME,XLAMUP

FORMAT(' '.5X.'FAIL='.F7.3,5X.'XK=',F7.3,5X,'TIME=',F7

2.3.5X.'XLAMUP = ',F10.6./)

TIME=TIME+20.0

IF(TIME.LE.51.0) GO TO 10
 101
                                IF(TIME.LE.51.0) GO TO 10
TIME=TIME+30.0
IF(TIME.LE.101.0) GO TO 10
 102
 103
                               IF(TIME.LE.101.0) GO
XK=XK+1.0
XKPRI=1.44015
IF(XK.LE.2.3) GO TO 7
XK=XK+2.0
XKPRI=1.403713
IF(XK.LT.5.5) GO TO 7
XK=XK+2.0
XKPRI=1.37725
IF(XK.LT.10.5) GO TO
XKPRI=1.37725
IF(XK.LT.10.5) GO TO
XKPRI=1.34092
IF(XK.LT.30.5) GO TO
STOP
END
 104
 105
 106
                                                                                                            GO TC 7
 107
                                                                                                         GO TC 7
                                END
```



COMPUTER PROGRAM FOR THE SEMI-CLASSIC METHOD

```
THIS SYSTEM IS LABELED "SEMI-CLASSIC" BECAUSE IT UTILIZES THE "PRIORS" THAT WERE INPUTS TO THE BAYES-IAN TECHNIQUE BUT THE COMPUTATIONS ARE PERFORMED THE THE SAME WAY AS IN THE "CLASSIC" TECHNIQUE.
 c
                                                                            THE VARIABLE DEFINITIONS FOR THIS PROGRAM FOLLOW:

XK
NUMBER CF COMPONENTS IN SERIES
NUMBER CF FAILURES PER COMPONENT
TIME NUMBER OF MISSION UNITS EACH COMPONENT
IS UNDER TEST

BETA OPTIMISTIC SCALE PARAMETER
ALFA OPTIMISTIC SHAPE PARAMETER
XLAMS SYSTEM FAILURE RATE
XLAMI ESTIMATE OF THE COMPONENT FAILURE RATE
XLAMUP UPPER CONFIDENCE LEVEL ON SYSTEM
FAILURE RATE
                                                           ## ALFA=0.5

XK=1.0

XKPRI=1.469496

## FAILURE RATE

ALFA=50.0

## TIME=30.0

C=0.0

XLAMUP=0.0

XLAMI=(ALFA+FAIL)/(TIME+BETA)

XLAMS=XK*XLAMI

C=(XK*XLAMI/(TIME+BETA)))/XLAMS

XLAMUP=((2.0*XLAMS)+((XKPRI***2)*C)+SORT(((4.0*XLAMS))*(XKPRI**2)*C)+SORT(((4.0*XLAMS))*(XKPRI**2)*C)+(XKPRI**4)*(**2)*/2.0

## RITE(6.101)ALFA-FAIL.XX.TIME.BETA,XLAMUP

## FORMAT(' '.2X.'ALFA=',F4.1.2X.'FAIL=',F4.1.2X,'XKE-'

## FOLOMOTION OF THE CONTROL OF THE 
 6
 7
89
 10
 101
  102
   103
  104
                                                                                   IF(ALFA.LE.1.1) GO TO 6
                                                                                   STOP
                                                                                   END
```



```
00000000000000
```

10

101

THIS PROGRAM IS A "MCDIFIED SEMI-CLASSIC" IN THAT THE FAILURES ARE MODIFIED SO THAT ONLY ONE COMPONENT OR ONLY TWO COMPONENTS FAIL DURING THE COMPLETE SYSTEM TEST. ALL OTHER VARIABLES ARE THE SAME AS FOR THE "SEMI-CLASSIC" PROGRAM.

ONLY ONE SET OF VARIABLES ARE TESTED WITH THIS PROGRAM. THE VARIABLES TIME, BETA, ALFA, AND FAIL WOULD HAVE TO BE VARIED TO GENERATE A FULL TABLE OF VALUES.

```
XKPRI=1.44015

XK=2.0

TIME=100.0

ALFA=0.0

ALFA=0.0

C=1.0/TIME

XLAMUP=(2.0~XLAMS+(XKPRI**2)~C+SQRT(((4.0*XLAMS)*(XKPRI**2)*C)+(XKPRI**4)*C**2))/2.0

WRITE(6.101)XK.BETA.TIME.XLAMUP

FORMAT(''.2X.'XK = '.F7.3.2X.'BETA = '.F7.3.2X.

2'TIME = '.F7.3.2X.'XLAMUP = '.F10.6./)

XK=XK+3.0

XLAMS=2.0/100.0

IF(XK.LE.5.5) GO TO 10

XK=XK+2.0

XLAMS=2.0/100.0

IF(XK.LE.10.5) GC TO 10

XK=XK+15.0

XLAMS=2.0/100.0

IF(XK.LE.31.0) GO TO 10

STOP

END
```



COMPUTER PROGRAM FOR THE BAYESIAN SIMULATION ALFA = 1.0

```
THE DEFINITIONS OF THE VARIABLES USED IN THIS
PROGRAM FOLLOW:
                                                         THE NUMBER OF MISSION UNITS THAT EACH COMPONENT IS UNDER TEST THE OPTIMISTIC SCALE PARAMETER THE OPTIMISTIC SHAPE PARAMETER THE NUMBER OF TIMES THAT EACH COMPONENT WILL FAIL DURING THE TEST TIME THE NUMBER OF COMPONENTS THAT ARE IN THE SERIES SYSTEM THE FAILURE RATE FOR AN INDIVIDUAL COMPONENT THE SYSTEM FAILURE RATE
                            TIME
                            BETA
                            ALFA
                            FAIL
                            K
                            XLAM
                            XLAMS
                  THIS SYSTEM IS CALLED DIMENSION XLAMS(1000).KEY(1000) TIME=30.0 BETA=50.0 FAIL=0.0 IND = 1 1000
            457
                  CONTINUE
      600
                                                = 1.1000
                                                = 0.0
      610
                  ALFA=1.0
B=TIME+BETA
KA=ALFA+FAIL
GO TO (601,602,603,604,606,607),IND
6
      601
                   GO TO 608
                   K = 2
GO TO
      602
                                   608
                  K = 5
GO TO 608
K = 10
GO TO:608
      603
      604
                  K = 30
IX=999
      606
       608
                         50 J=1.1000
609 ILBDS =
9
                   DO
               DD 609 ILBDS = 1.K

TR=1.0

DD 20 I=1.KA

CALL RANDU(IX.IY.YFL)

IX=IY

TR=TR*YFL

XLAM=-ALOG(TR)/B

XLAMS(J) = XLAMS(J) + XLAM

CONTINUE

KEY(J)=J

CONTINUE

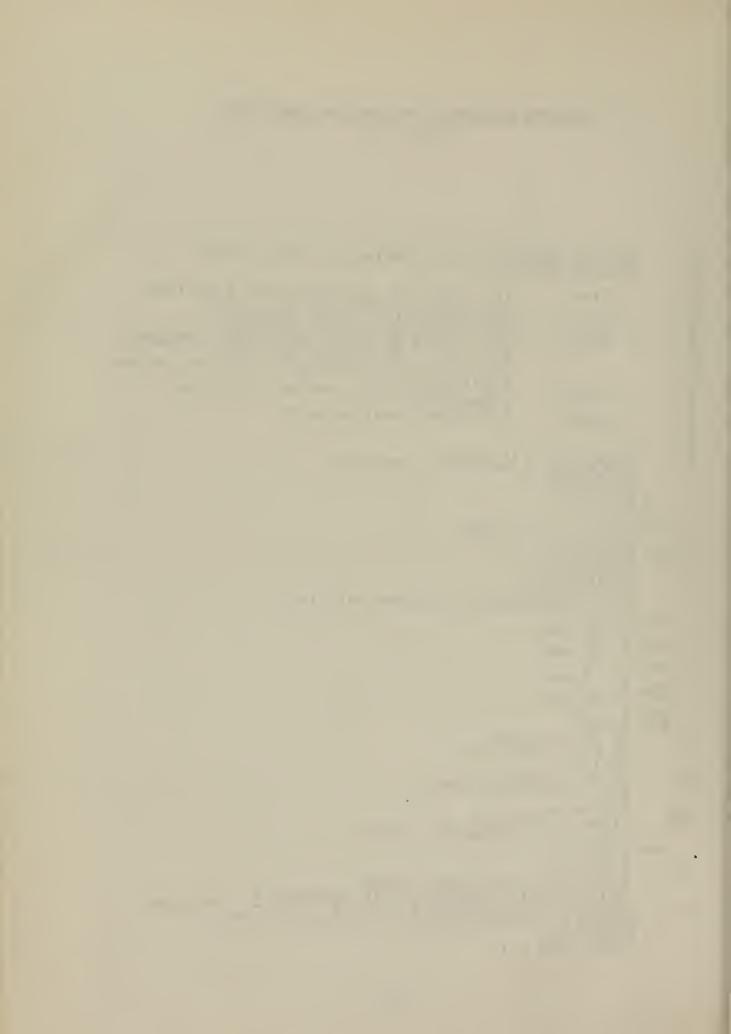
CALL SHSORT(XLAMS.KEY.1000)

WRITE(6.60)K.TIME.BETA.FAIL.XLAMS(900)

FORMAT(' '.3X.'COMP. = '.I4.3X.'TIME = '.F7.2.3X.

1'BETA = '.F7.2.3X.'FAIL = '.F6.2.3X.'LAMS = '.

2F10.6.//)
                   DO
 10
 20
       609
 50
 60
                2F10.6.//)
IND = IND +
```

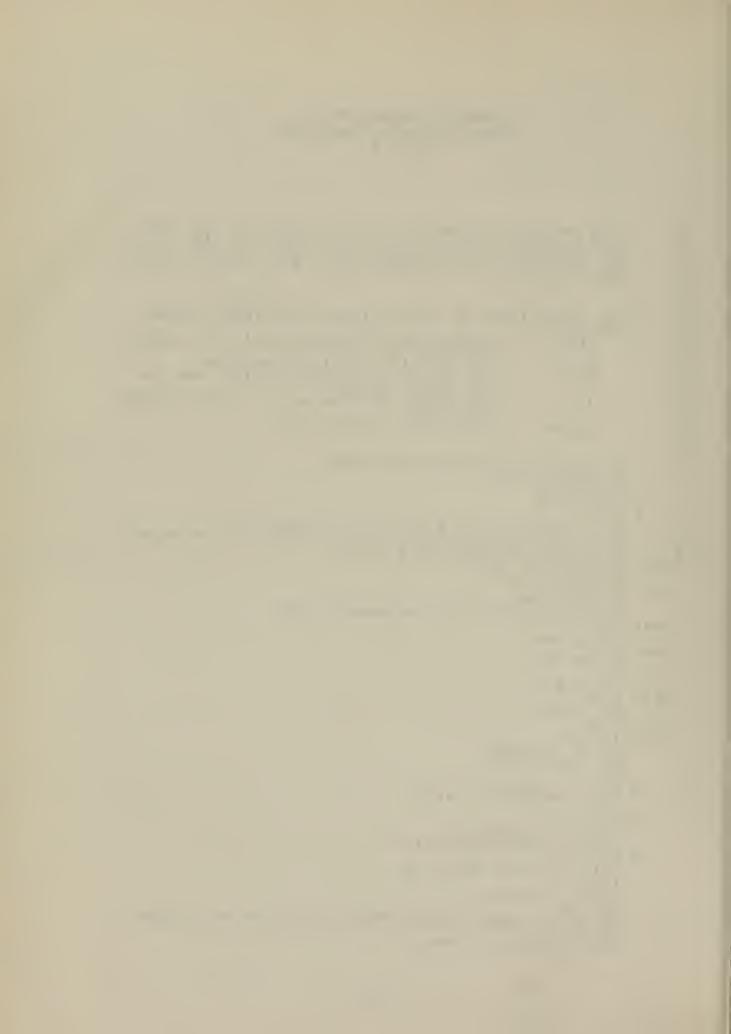


GO TO 600
6C7 CGNTINUE
FAIL=FAIL+1.0
IF(FAIL.LE.1.5) GO TO 7
BETA=BETA+50.0
IF(BETA.LE.100.1) GO TO 5
TIME=TIME+20.0
IF(TIME.LE.50.1) GO TO 4
TIME=TIME+30.0
IF(TIME.LE.100.1) GO TO 4
STOP
END



COMPUTER PROGRAM FOR THE MODIFIED BAYESIAN SIMULATION ALFA = 1.0

```
MUCH THE
WAS IN THAT
THAT ONLY
THAT THE SYS-
            THIS BAYESIAN TECHNIQUE IS "MODIFIED" IN SAME WAY THAT THE SEMI-CLASSIC TECHNIQUE THE NUMBER OF FAILURES ARE "MODIFIED" SO ONE AND ONLY TWO COMPONENTS WILL FAIL DU
DURING
                     TEST
             THE DEFINITIONS OF THE VARIABLES FOR THIS PROGRAM
            FOLLOW:
TIME
                                        THE NUMBER OF MISSION UNITS THAT EACH COMPONENT IS UNDER TEST THE OPTIMISTIC SCALE PARAMETER THE NUMBER OF COMPONENTS THAT ARE IN
                   BETA
                                               NUMBER
SERIFS
NUMBER
                   K
                                                               SYSTEM
OF COMPONENTS IN THE SYSTEM
                                        THE
                   JJ
                                                 FAIL
SYSTEM
                                        THAT
                   XLAMS
                                        THE
                                                               FAILURE RATE
            DIMENSION XLAMS(1000), KEY(1000)
DO 700 JJ=1,2
             TIME=30.0
             BFTA = 50.0
            KA IS EQUAL TO ALFA PLUS FAIL,
THE INDEX "JJ" IS EQUAL TO THE
THAT FAIL IN THE SERIES SYSTEM
DO 610 IC=1.1000
XLAMS(IC)=0.0
CCNT INUE
B=REIALTIME
000
                                                                              WHICH IS
                                                                              WHICH IS 2
NUMBER OF COMPONENTS
    600
    610
             B=BETA+TIME
            GO TO (60
K=1
GO TO 608
                   TO (601,602,603,604,606,607),IND
    601
            K=2
GO TO 608
    602
            GO K=5
    603
             GO TO 608
            GU 10 808
K=10
GO TO 608
K=30
IX=999
DO 80 J=1,1000
DO 24 JKJ=1,JJ
    604
    606
608
            DU 24 JKJ=1,JJ
TR=1.0
DO 20 I=1.2
CALL RANDU(IX.IY.YFL)
IX=IY
TR=TR*YFL
XLAMX=-ALOG(TR)/B
XLAMX(J)=XLAMS(J)+XLAMX
CONTINUE
      20
             IF(K-JJ.LE.O) GO TO 79
             L=K-JJ
DO 30 ILBDS=1,L
             TR=1.0
KA IS NOW EQUAL TO ONE FOR THE REST OF THE SYSTEM DO 25 II=1.1
C
             CALL RANDU(ÎX, IY, YFL)
```



```
IX=IY

25 TR=TR*YFL
    XLAMS(J)=XLAMS(J)+XLAM

30 CONTINUE

79 KEY(J)=J

80 CCNTINUE
    CALL SHSORT(XLAMS,KEY,1000)
    WRITE(6.90)K.TIME.BETA,XLAMS(900)

90 FORMAT(''.3X,'CCMP. = '.I4,3X.'TIME = ',F7.2,3X,
    *'BETA = '.F7.2.3X,'XLAMS = ',F10.6,//)

IND=IND+1
    GO TO 600

607 CONTINUE
    BETA=BETA+50.0
    IF(BETA-LE.100.1) GO TO 5
    TIME=TIME+20.0
    IF(TIME.LE.50.1) GO TO 4

TIME=TIME+30.0
    IF(TIME.LE.100.1) GO TO 4

760 CCNTINUE
    STOP
    END
```



COMPUTER PROGRAM FOR THE BAYESIAN SIMULATION ALFA = 0.5

```
THIS PROGRAM WILL GENERATE VARIATES FROM THE GAMMA DISTRIBUTION WHEN THE SHAPE PARAMETER (ALFA) IS LE
                THAN 1.0
               THE VARIABLE DEFINITIONS FOR THE MAIN PROGRAM FOLLOW:
                                                 THE OPTIMISTIC SHAPE PARAMETER
THE OPTIMISTIC SCALE PARAMETER
THE NUMBER OF MISSION UNITS THAT EACH
COMPONENT IS UNDER TEST
THE NUMBER OF TIMES THAT EACH COMPONENT
FAILS CURING THE TEST TIME
THE NUMBER OF COMPONENTS IN THE SERIES
                       XBETA
                       XTIME
                       XFAIL
                                                 THE NUMBER OF COMPONENTS IN THE S
SYSTEM
THE SYSTEM FAILURE RATE
THE UPPER CONFIDENCE LEVEL ON THE
                       NC MP
                       XLAMS
                       XLAMUP
                                                 FAILURE RATE
               DIMENSION XLAMS(1000), KEY(1000)
               REAL*4 K
K=0.5
XTIME=30.0
               XBETA=50.0
       30
               IND=1
CO 610 IO=1,1000
XLAMS(IO)=0.0
        40
     600
     610
               CONTINUE
               BETA=1.0/(XBETA+XTIME)
GO TO(601.602.603.604.605.606),IND
               NCMP = 1
     601
                GO TO 607
               NCMP=2
GO TO 607
NCMP=5
     602
     603
               GO TO 607
               NCMP=10
     604
               GO TO 607
NCMP=30
IX=999
     605
            IX=999

CALL GMINIT(K,BETA)

DO 50 J=1.1000

CO 609 ILBCS=1.NCMP

CALL GAMA(IX.Z)

Z WILL = AN UNORDERED VALUE OF LAMBDAI

XLAMS(J)=XLAMS(J)+Z

CCNTINUE

KEY(J)=J

CONTINUE

NOW ORDER THE SYSTEM FAILURE RATES

CALL SHSORT(XLAMS.KEY.1000)

WRITE(6.60)NCMP.XBETA.XTIME.XLAMS(900)

FORMAT(' '.2X.'NCMP ='.14.2X.'BETA ='.F6.2.2X,

*'TIME ='.F6.2.2X.'XLAMS ='.F10.6,/)

IND=IND+1

GO TO 600
     607
        10
C
     609
C
               GO TO 600
CONT INUE
XBETA=XBETA+50.0
     606
```



IF(XBETA.LE.100.1) GO TO 40 XTIME=XTIME+20.0 IF(XTIME.LE.50.1) GO TO 30 XTIME=XTIME+30.0 IF(XTIME.LE.100.1) GO TO 30 END



SUBROUTINE GMINIT (K, BETA)

ADDITIONAL ENTRY POINTS:

RSULT GAMA(IX.Z)

PURPOSE:

GENERATION OF GAMMA PARAMETER LESS THAN RANDOM DEVIATES WITH SHAPE ONE.

.METHOD:

A MODIFICATION OF MARSAGLIA'S BOX-WEDGE-TAIL METHOD FOR NORMAL DEVIATES IS USED. THE PDF IS DECOMPOSED INTO A HEAD REGION. A NUMBER (DEPENDENT ON K) OF RECTANGLES AND WEDGES AND A TAIL REGION. THE GMINIT SECTION OF THE SUBROUTINE ALSO SETS UP A BINARY SEARCH TREE TO BE USED FOR EFFICIENT SELECTION OF TPROPER REGION DURING THE ACTUAL GENERATING PROCESS. WHICH IS HANDLED BY THE GAMA SECTION TAIL METHOD DECOMPOSED ON K) OF THE GMINIT

DESCRIPTION OF PARAMETERS

GAMMA DISTRIBUTION SHAPE PARAMETER (MUST BE .GE. 0.05 AND .LE. 1.0)
GAMMA DISTRIBUTION SCALE PARAMETER SEED FOR RANDEM NUMBER GENERATOR RETURNED GAMMA DEVIATE K BETA ΪΧ Ζ

THE PDF OF THE GAMMA FUNCTION IS GIVEN BY

 $F(X) = (1/BETA) \times K \times X \times (K-1) \times EXP(-X/BETA)/GAMMA(K)$

THE FOLLOWING SUBROUTINES ARE USED:

RETURNS A UNIFORM (0,1) DEVIAT COMPUTES THE INVERSE GAMMA CDF COMPUTES THE INCOMPLETE GAMMA FUNCTION (GAMMA CDF) RANDOM(IX)
INVGAM(K,X)
IGAM(K,X) DEVIATE

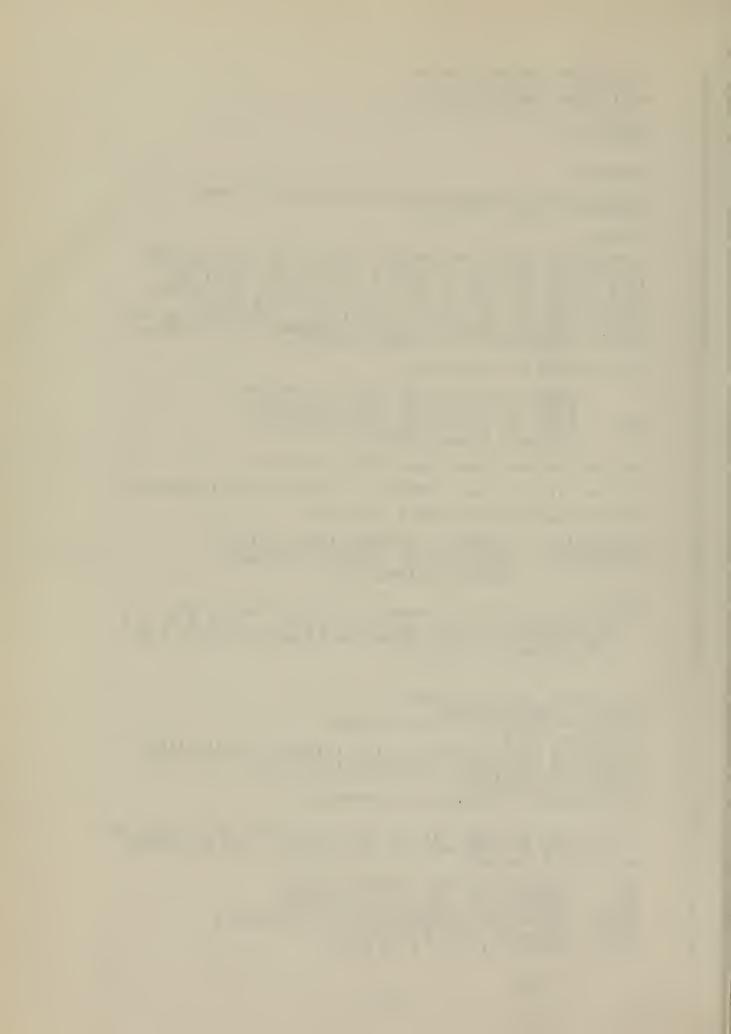
NOTE: UNDERFLOW IS POSSIBLE WHEN K IS LESS THAN.18 AND BECOMES MORE LIKELY AS K DECREASES. WHEN K IS O THE PROBABILITY OF UNDERFLOW IS ABOUT.0C0129 FOR ANY GENERATED DEVIATE.

SUBROUTINE GMINIT(K.BETA)
REAL*4 K,INVGAM.IGAM
INTEGER*4 FIRST,TABLE,BOTTOM.END
LOGICAL*1 USED
DIMENSION P(100).X(101).H(100).Q(100).R(100).B(100)
DIMENSION TABLE(202).PROB(202).NEXT(202).LAST(202)
DIMENSION TEST(202).LIST(202).USED(202)
DIMENSION RAND(2)
EQUIVALENCE (U.RAND(1)).(V,RAND(2))

THIS FIRST SECTION
BE USED BY GAMA
ARE USED BY GAMA: INITIALIZES CONSTANTS A WHEN IT IS CALLED. THE AND TABLES

PROBABILITY FOR "HEAD" REGION
PROBABILITY FOR "TAIL" REGION
PROBABILITY FOR I-TH RECTANGLE
LEFT-HAND BCUNDARY OF I-TH RECTANGLE
WIDTH OF I-TH RECTANGLE
PROBABILITY OF I-TH WEDGE PO PN P(I) X(I) H(I)

0(I)



```
REJECTION TEST RATIO FOR I-TH WEDGE
Y INTERCEPT FOR I-TH WEDGE
SHAPE PARAMETER - 1.
ORDERED VECTOR OF PROBABILITIES
VECTOR OF WEDGE/RECTANGLE NUMBERS CCRRESPOND-
ING TO PROBABILITIESS IN PROB
STARTING POINT FOR BINARY SEARCH
LINKS FOR BINARY SEARCH
            R(I)
B(I)
ALPHA
PROB
             TABLE
            FIRST
NEXT.
LAST
                         POSITION IN PROB OF PO
CONSTANTS FOR APPROXIMATION TO INVERSE GAMMA
CDF FOR SMALL VALUES OF Z
             J1
H1 TO
              H4
           CHECK FOR K IN RANGE
IF((K.GE.O.O5) .AND. (K.LE.1.0))GO TO 5
WRITE(6.4)K
FORMAT(//'JGMINIT CALLED WITH K=',1PE16.6,
                   CUT
                           OF RANGE 1/)
             RETURN
             GET UPPER BOUND ON NUMBER OF RECTANGLES
            N=20.+6.6/K
IF(N.GT.100)N=100
M=2*N+2
             MM = M - 1
            ALPHA=K-1.

GK=GAMMA(K)

PO=5.E-5/(K*K)

HFAC=2.
             SET UP RECTANGLE BOUNDS
             X(1) = INVGAM(K, PO)
            X(I)=INVGAM(K.FO)
P0=IGAM(K.X(1))
H(1)=.25*X(1)
C0 10 I=2.N
X(I)=X(I-1)+H(I-1)
H(I)=H(I-1)*HFAC
P(I)=0.
            O(1)=0.
CCNTINUE
X(N+1)=X(N)+H(N)
CC
      ZERO PROBABILITY VECTORS AND LINKS
DO 15 I=1.M

NEXT(I)=0

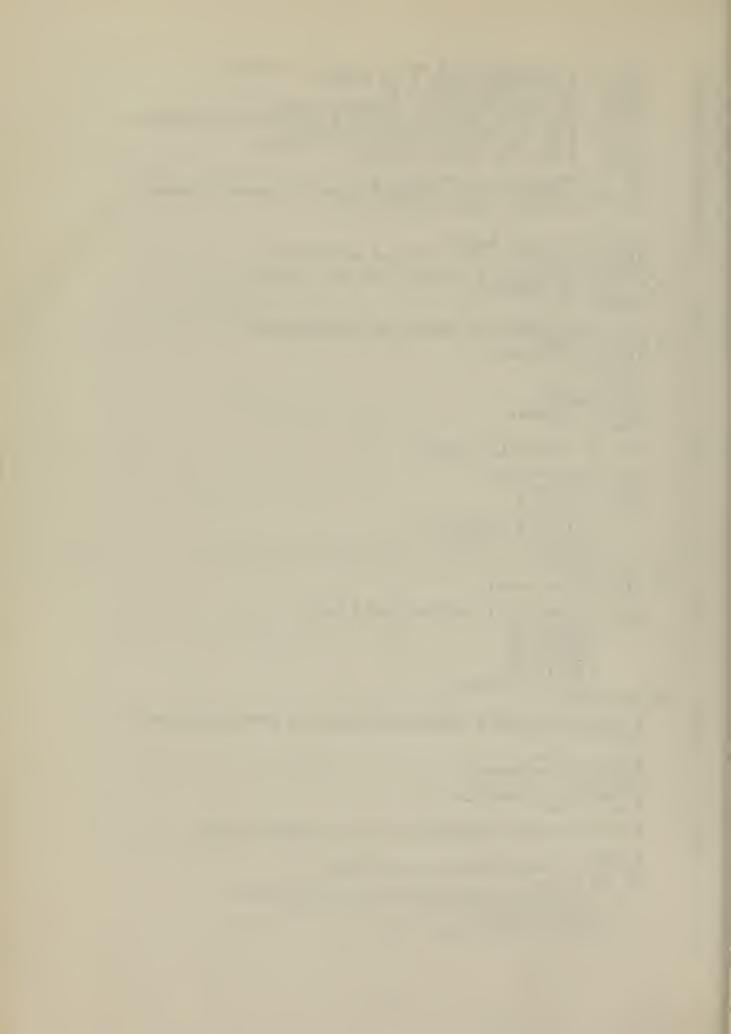
LAST(I)=0

PROB(I)=0.

LIST(I)=0

USED(I)=.FALSE.

15 CONTINUE
CCCC
             FIND COEFFICIENTS FOR NEWTON-RAPHSON APPROXIMATION TO SLOPE OF PDF
             B1=-2. * ALPHA
B2=ALPHA * (ALPHA-1.)
             A1=B1+1.
A2=ALPHA*(ALPHA-2.)
             C=1.-ALPHA
             FIND RECTANGLE PROBABILITIES AND WEDGE VALUES
             PL=P0
             FL=EXP(ALPHA*ALOG(X(1))-X(1))/GK
                       I=1.N
FU=EXP(ALPHA*ALOG(X(I+1))-X(I+1))/GK
PU=IGAM(K.X(I+1))
P(I)=H(I)*FU
             DG 40
                        O(I) = PU - PL - P(I)
C
```



```
NEWTON-RAPHSON ITERATION TO FIND POINT WHERE SLOPE OF PDF IS PARALLEL TO CHORD
                 M = X(I)
                 W=X(I)

S=(FU-FL)/H(I)

SC=S*GK

DO 20 J=1.8

Y=W*((W+A1)*W+A2+SC*EXP(C*ALOG(W)+W))/((W+B1

IF(ABS(Y-W).LT.1.E-4*Y)GO TO 30
     20
                 CONTINUE
000
                 FIND VALUES FOR REJECTION METHOD
                 B(I)=EXP(ALPHA*ALOG(Y)-Y)/GK+S*(X(I)-Y)
     30
                 R(I)=(B(I)-FU)/(FL-FU)
C
                 TEST TO SEE IF ENOUGH RECTANGLES HAVE BEEN TAKEN IF (PU.GT. 0.999) GO TO 45
                 RESET PROBABILITY AND FUNCTION VALUES FOR NEXT
                 RECTANGLE
                 PL=PU
                 FL=FU
    40 CONTINUE
C
         FIND LOWER END OF NCN-ZERO PROBABILITY VECTOR LOW=2*(N-I)+1
     45
CC
         FIND TAIL PROBABILITY SUM1=PO
         DO 50 J=1.N
SUM1=SUM1+P(J)+Q(J)
         CONTINUE
         PN=1.-SUM1
0000
         GET VALUES TO COMPUTE LOW END APPROXIMATION TO INVERSE
         GAMMA CDF
         H1=K±GK×P0
         H2=1./K
         H3=(K+1.)*5.0E-7
H4=4./(K+1.)
CCC
         GENERATE PROBABILITY VECTOR
         DO 80 I=1.N

PROB(I)=P(I)

TABLE(I)=I

PROB(I+N)=Q(I)

TABLE(I+N)=-I
         CONT INUE
PROB (M-1)=PO
TABLE (M-1)=0
PROB (M)=PN
     80
         TABLE (M) = 0
           SORT PROBABILITY VECTOR
         DO 110 I = 1 . MM
                 ICH=O
L=M-I
                         0 J=1.L
IF(PROB(J)-PROB(J+1))100,100,90
                 DO
                      100
                         TEMP=PROB(J)

PROB(J)=PROB(J+1)

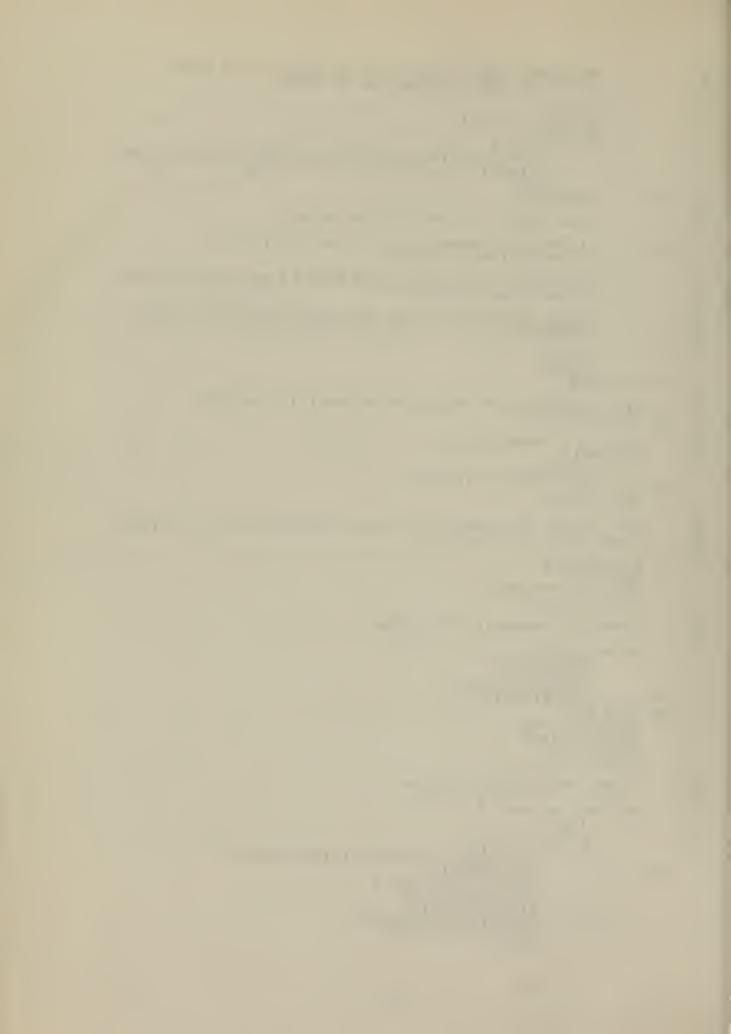
PROB(J+1)=TEMP

ITEMP=TABLE(J)

TABLE(J)=TABLE(J+1)

TABLE(J+1)=ITEMP

ICH=1
     90
```

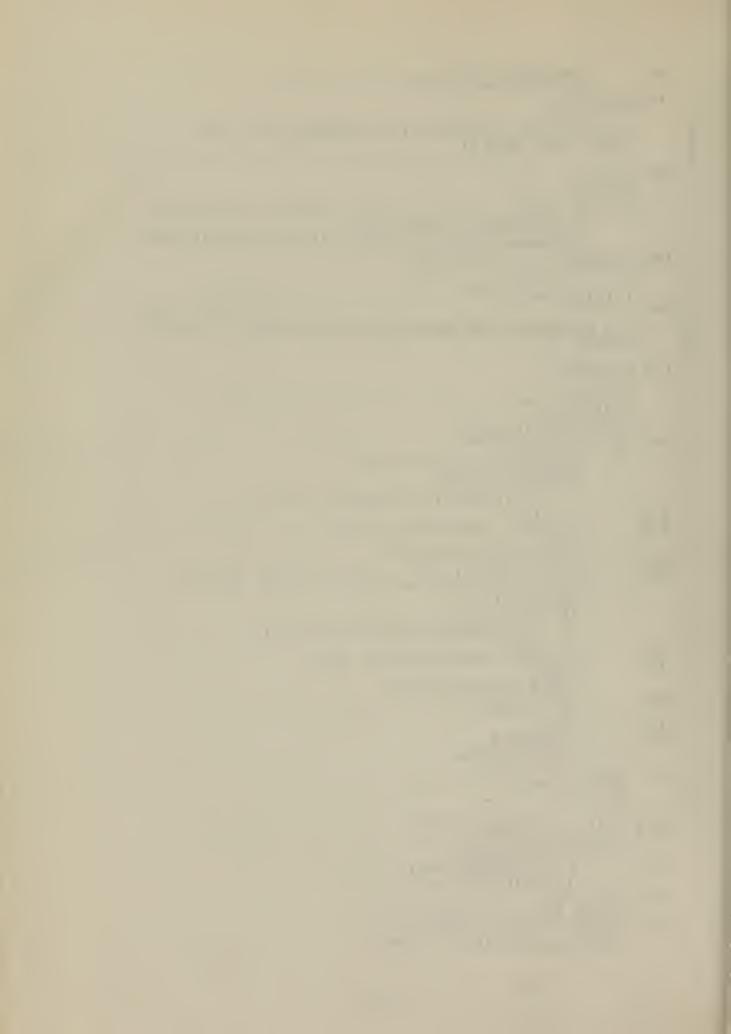


```
CONTINUE
  100
  1F(ICH)120,120,110
110 CCNTINUE
        PROB(M) = 1.
0000
       CCNVFRT PROB TO CUMULATIVE PROBABILITIES AND
         FIND FIRST AND J1
       120
  130 CONTINUE
C
       J1=LOW
  140
0000
       NOW DETERMINE THE VECTORS NEXT AND LAST FOR BINARY
        SEARCH
  150
        BOTTOM=1
      151
  152
153
              J=J-1

IF(LI-J)153.155.155

NEXT(LI)=J

LIST(I)=NEXT(LI)
  154
155
              IF((LI.EO.LOW) .OR. USED(LI-1))G
PRL=TEST(I)+PR
TEST(I)=PRN
DO 156 J=LOW.MM
IF(PROB(J).GT.PRL)GO TO 157
                                 .OR. USED(LI-1))GO TO 159
  156
157
              CONTINUE
             IF(.NOT. USED(J))GC TO 1571
J=J+1 ·
IF(LI-J)158,158,157
·LAST(LI)=J
 1571
              GO TO 1585
J=LI
 158
1585
       END=END+1
LIST(END)=J
TEST(END)=PRL
CONTINUE
BOTTCM=END
PR=PR#20 5
  159
        PR=PR*3.5
       1591
   160
  161
  162
        GO TO 1591
USED(LIST(I))=.TRUE.
  163
        I=I+1
IF(I.LE.BOTTOM)GO TO 1591
```



```
END=BOTTCM
            GO TO 151
             SETUP FIRST CALL TO RANDOM
    165 CALL OVFLOW
00 0000
            THROUGH WITH SETUP PHASE. QUIT.
             RETURN
            THIS SECTION PRINTS OUT THE VALUES GENERATED BY GMINIT
         ENTRY RSULT
WRITF(6,166)K,BETA

FORMAT(/'IGENERATED VALUES FOR K=',1PE14.6,

*' BETA=',E14.6)
WRITE(6,170)PO.PN

FORMAT('OHEAD PROBABILITY=',1PE15.6,

*' TAIL PROBABILITY=',E15.6)
WRITE(6,18J)
FORMAT('ORECTANGLE/WEDGE VALUES'//2X,'I',9X,'X(I)',12X

*'P(I)',12X,'Q(I)',12X,'R(I)',12X,'B(I)'/)
SUM1=0.
             ENTRY RSULT
    166
    180
             SUM 1 = 0 .
            SUM1=0.

SUM2=0.

CO 192 I=1.N

IF(P(I))193,193,185

SUM1=SUM1+P(I)

SUM2=SUM2+Q(I)

WRITE(6.190)I.X(I).P(I).Q(I).R(I).B(I)

FORMAT(1XI3.1P5E16.6)
    185
    190
192
          CONTINUE

WRITE(6.194)SUM1.SUM2

FORMAT('OSUMS'.15X.1P2E16.6)

WRITE(6.195)J1.H1,H2.H3.H4

FORMAT(/'OVALUES FOR HEAD/TAIL APPROXIMATION:'/'

* I6/' H1='.E16.6.' F2='.E16.6.' H3='.E16.6.'
    193
    194
                                                                                                                    J1='
                                                                                                                       H4 1 .
          1E16.6)

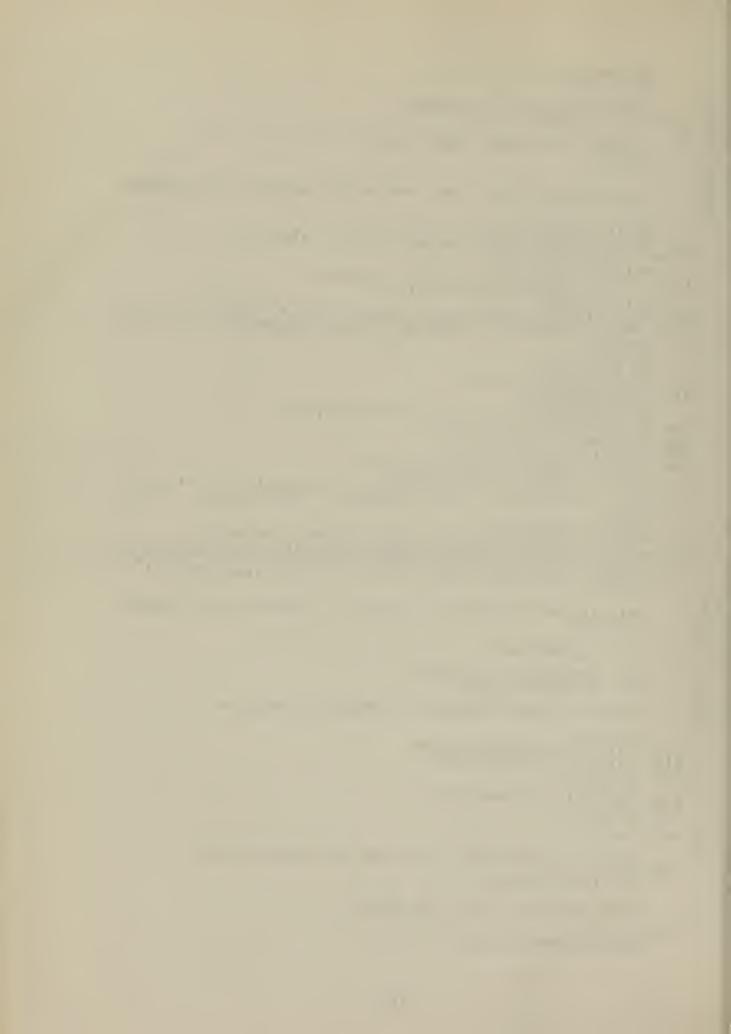
WRITE(6.196)FIRST

FORMAT(/'OSTARTING PCINT FOR BINARY SEARCH', I4)

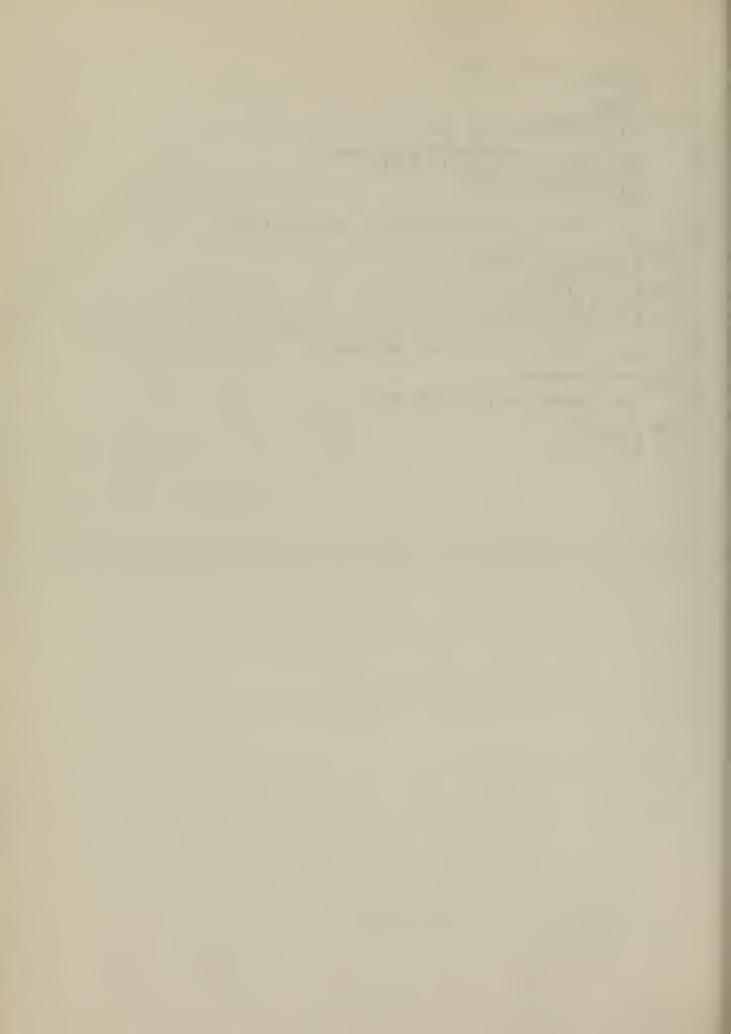
WRITE(6,197)(I, PROB(I), TABLE(I), NEXT(I), LAST(I), I=1, M)

FORMAT(/'OPROBABILITY VECTOR AND LINKS'//14X, 'PROB', 9X

*.'TABLE', 4X, 'NEXT', 5X, 'LAST'//(1XI5, 1PE16.6, JP3I9))
    196
             RETURN
0000
            THIS IS VARIATES
                            THE SECTION THAT ACTUALLY COMPUTES THE RANDOM
            ENTRY GAMA(IX. Z)
             GET TWO UNIFORM DEVIATES CALL RANDOM (IX. U.2)
             CONDUCT BINARY SEARCH OF PROBABILITY VECTOR
             J=FIRST
            IF(U-PROB(J))210,250,230
IF(LAST(J))250,250,220
    200
    210
            J=LAST(J)
GO TO 200
IF(NEXT(J))245,245,240
J=NEXT(J)
GO TO 200
    220
    230
    240
    245
             J=J+1
             LOCATED PROBABILITY DIVISION. GET TABLE VALUE.
            N=TABLE(J)
    250
             IF(N)260,290,320
             THIS SECTION IS FOR THE WEDGES
    260
           N = -N
             CALL RANDOM(IX,U,1)
```



```
270
          IF(U.LE.V)GO TO 280
          TEMP = U
U=V
          V=TEMP
Z=X(N)+H(N)*U
IF(V.LE.R(N))GO TO 330
   280
CC
          THIS STEP IS PERFORMED VERY RARELY W=U+EXP(ALPHA*ALOG(Z)-Z)/B(N) IF(V.LE.W)GO TO 330 CALL RANDOM(IX.U.2) GO TO 270
000
          THIS SECTION IS FOR HEAD/TAIL PROBABILITIES
          IF(J.EQ.J1)GO TO 300
Z=INVGAM(K.1.-PN*V)
GO TO 330
Z=(H1*V)**H2
   290
   300
          IF(Z.LT.H3)GO TO 330
Z=2.*Z/(1.+SORT(1.-H4*Z))
GO TO 330
000
          THIS SECTION IS FOR THE RECTANGLES
   320 Z=X(N)+H(N)*V
          SCALE GAMMA VARIATE AND EXIT
          Z=Z*BETA
RETURN
END
   330
```



INVGAM SUBROUTINE

C

C

```
FUNCTION INVGAM(K,Z)
REAL*4 INVGAM,IGAM,K,EPS/1.E-08/
IF(Z.GT.0.0)GO TO 10
INVGAM=0.0
RETURN

10 X=Z
T=K-1.
G=GAMMA(K)
DO 40 I=1.30
Y=G*(IGAM(K.X)-Z)/(EXP(T*ALOG(X)-X))

20 IF(Y.LT.X)GO TO 30
Y=Y*.5
GO TO 20

30 X=X-Y
IF(ABS(Y)-EPS*X)50.5C.40

40 CONTINUE
50 INVGAM=X
RETURN
THIS IS THE END OF THE INVGAM SUBROUTINE
END
```

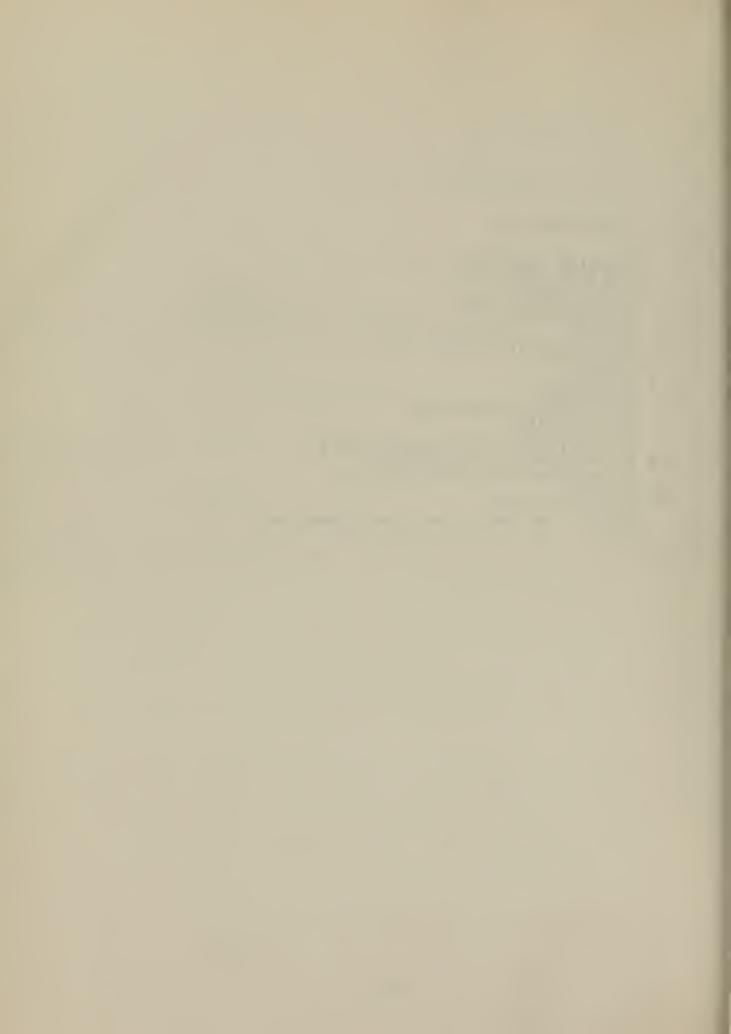


```
FUNCTION IGAM(K.X)
IMPLICIT REAL*8 (D)
REAL*4 IGAM.K
REAL*8 EPS/1.D-13/
IF(X.GT.J.)GO TO 5
IGAM=0.
RETURN
5 IF(X.LE.12.0)GO TO 8
IGAM=1.0
RETURN
8 DX=DBLE(X)
CK=DBLE(K)
DTERM=DX**DK/DGAMMA(CK)
DSUM=DTERM/CK
CO 20 I=1.30
IF(DABS(DTERM)-EPS*DSUM)40.40.10
10 DTERM=DTERM*(-DX)/DFLOAT(I)
CSUM=DSUM+DTERM*(-DX)/DFLOAT(I)
20 CONTINUE
40 IGAM=SNGL(DSUM)
RETURN
THIS IS THE END OF THE IGAM SUBROUTINE
```

IGAM SUBROUTINE

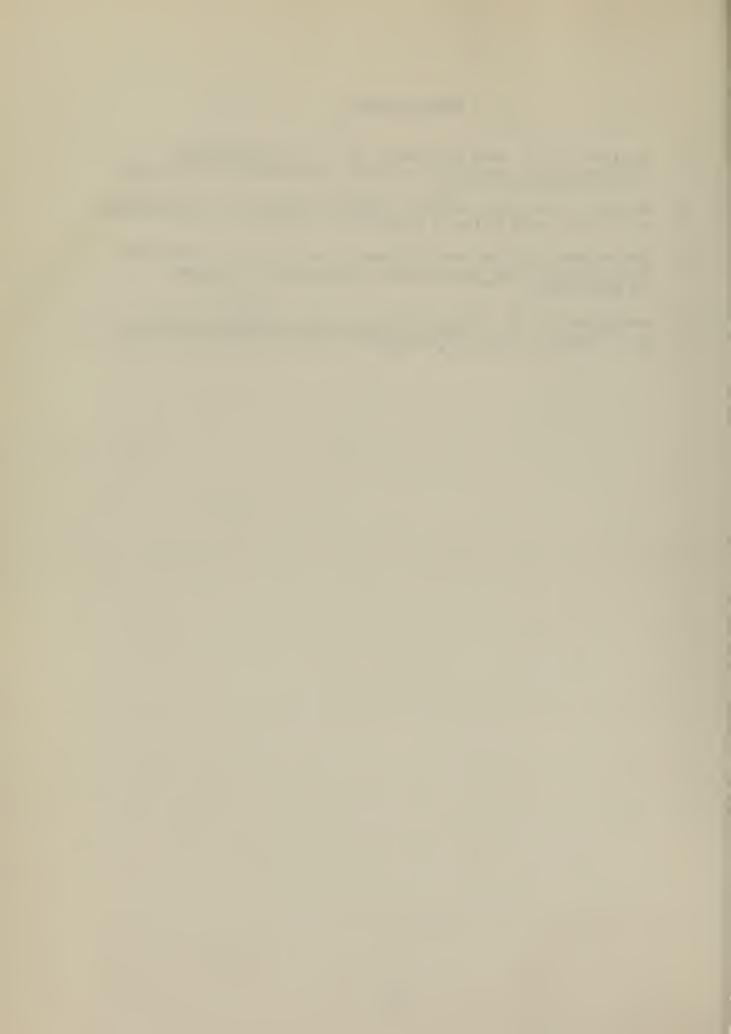
END

C



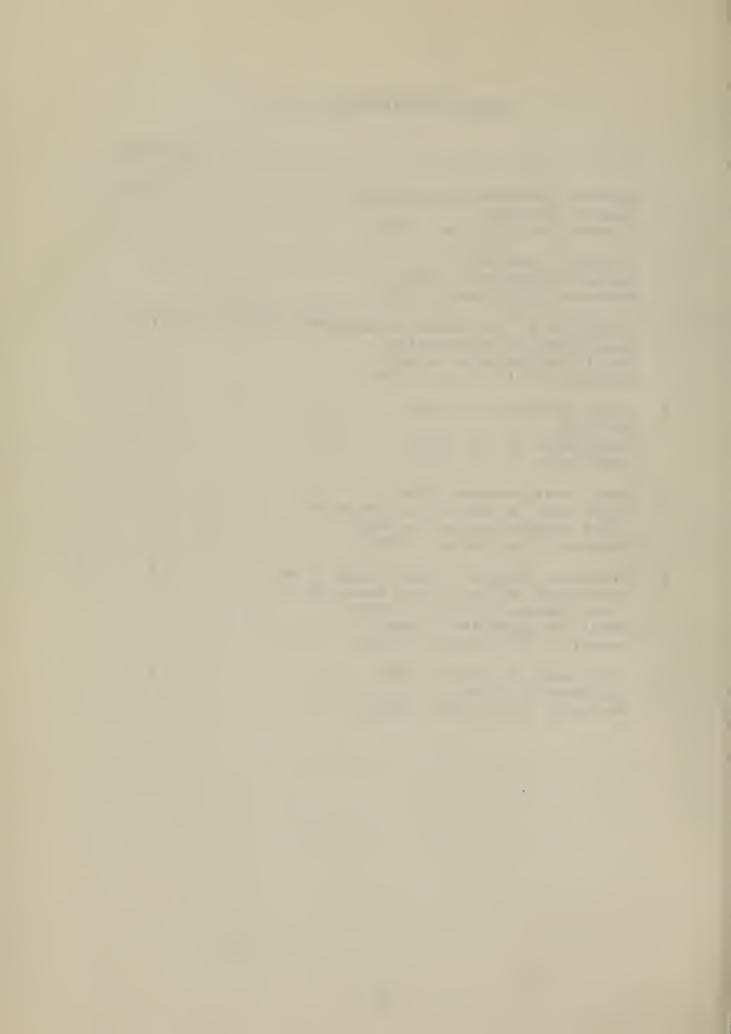
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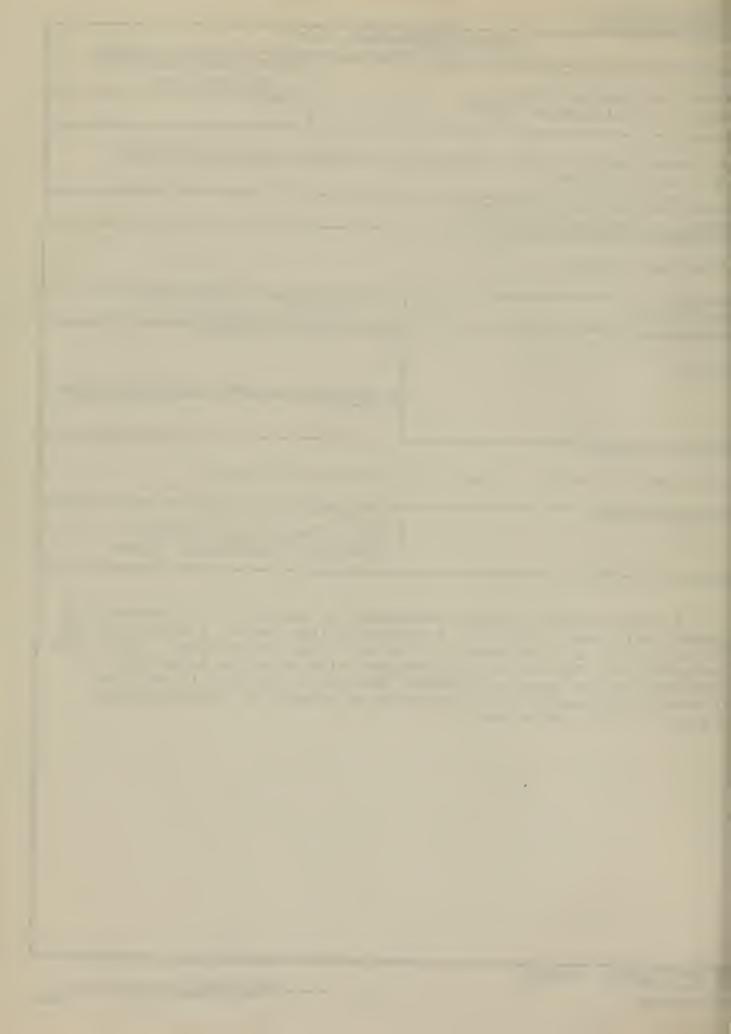
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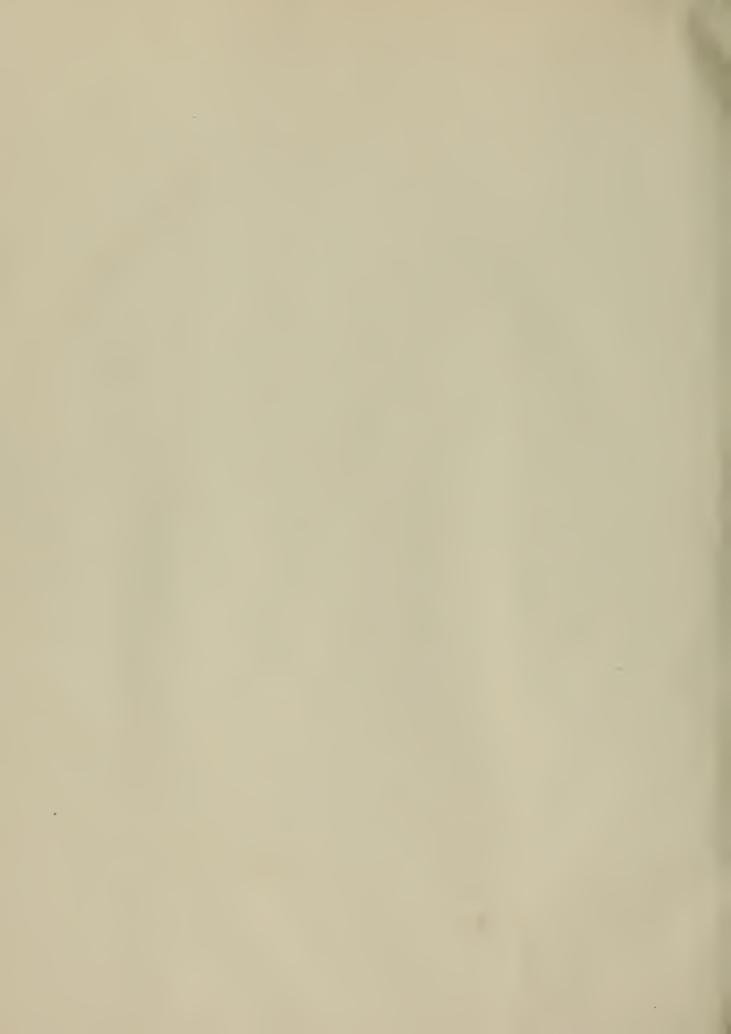
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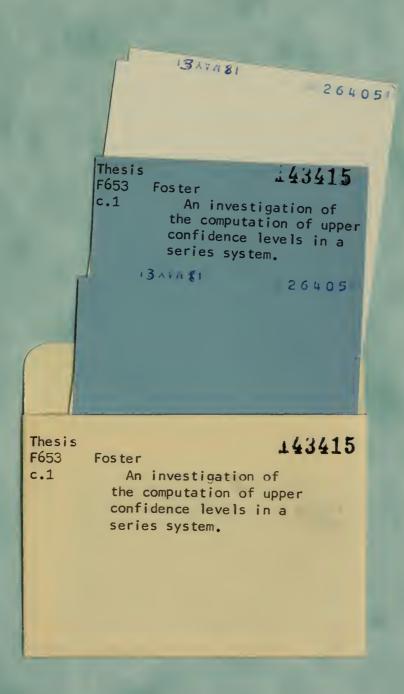
J. ABSTRACT

A comparison of several techniques is presented for determi ing upper confidence levels for a system failure rate. A scries sy tem of components with exponential failure rates is examined. Classical computational techniques are compared with Bayesian techniques in determining the upper confidence level of a system failure rate. A sensitivity analysis is conducted on several of the parameters as part of the comparison.



UNCLASSIFIE) Security Classification LINK A LINK B LINK C KEY WORDS ROLE ROLE ROLE Gamma Function Bayesian Series System Reliability Upper Confidence Level Reliability Apriori Aposteriori Exponential





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